# Journées "Graphes & surfaces"

# les 1, 2, 3 février 2016

# à Grenoble

G-SCOP - 46 Avenue Félix Viallet - salle C319

Accès: http://www.grenoble-inp.fr/acces-itineraires/laboratoire-g-scop-575967.kjsp?RH=GSCOP FR-MAP

Les exposés durent 1h et seront entourés de discussions informelles.

# Lundi 1er février

- 10h Louis Esperet Box representations of embedded graphs
- 14h Torsten Ueckerdt Interval Representations of Graphs
- 16h Nicolas Bonichon On the enumeration of planar Eulerian orientations

## Mardi 2 février

- 10h Marie Albenque TBA
- 14h Benjamin Lévêque Encoding toroidal triangulations
- 16h Arnaud De Mesmay Shortest path embeddings of graphs on surfaces

# Mercredi 3 février

- 9h30 Daniel Gonçalves 3-colorable planar graphs are 3-DIR
- 11h Vincent Despré Computing the geometric intersection number of curves
- 14h Matej Stehlik Matej Stehlik Edge- and face-width of projective quadrangulations

#### Vincent Despré - Computing the geometric intersection number of curves

The geometric intersection number of a curve on a surface is the minimal number of self-intersections of any homotopic curve, i.e. of any curve obtained by continuous deformation. Likewise, the geometric intersection number of a pair of curves is the minimal number of intersections of any homotopic pair. Given two curves represented by closed walks of length at most  $\ell$  on a combinatorial surface of complexity n we describe simple algorithms to compute the geometric intersection number of each curve or of the two curves in  $O(n+\ell 2)$  time. We also propose an algorithm of complexity  $O(n+\ell \log 2\ell)$  to decide if the geometric intersection number of a curve is zero, i.e. if the curve is homotopic to a simple curve.

#### **Torsten Ueckerdt - Interval Representations of Graphs**

An interval graph is the intersection graph of a discrete set of intervals on the real line. Here we consider two natural generalizations of interval graphs: Given any graph, we want to represent it as the union or the intersection of several interval graphs. The first goal is to minimize the number of interval graphs used in the representation. But we will also discuss the local and folded variants of this representation problem. On the way, we shall discuss a new proof that the interval number of any planar graph is at most three---a result originally claimed in 1982.

#### Nicolas Bonichon - On the enumeration of planar Eulerian orientations

Eulerian planar maps are planar maps with all vertices of even degree. The number of Eulerian planar maps with a fixed number of edges m is well known, and they are in bijection with many combinatorial families, such as balanced Eulerian trees or bi-cubic maps. Planar Eulerian orientations are directed planar maps in which the in- and out-degrees of every vertex are equal.

We first count the number of planar Eulerian orientations for  $m \le 12$  (m is the number of edges) thanks to an encoding based on bilateral Dyck words. The values found (2, 10, 66, 504, 4216, 37548...) do not match with any known series from the OEIS.

We then present a sequence of algebraic series giving lower and upper bounds of the growth rate of planar Eulerian orientations. We show in particular that the growth rate of planar Eulerian orientations is between 10.69m and 12.94m.

### Marie Albenque - TBA

#### Benjamin Lévêque - Encoding toroidal triangulations

Poulalhon and Schaeffer introduced an elegant method to linearly encode a planar triangulation optimally. The method is based on performing a special depth-first search algorithm on a particular orientation of the triangulation: the minimal Schnyder wood. Recent progress toward generalizing Schnyder woods to higher genus enables us to generalize this method to the toroidal case. In the plane, the method leads to a bijection between planar triangulations and some particular trees. For the torus we obtain a similar bijection but with particular unicellular maps (maps with only one face).

#### Arnaud De Mesmay - Shortest path embeddings of graphs on surfaces

The classical theorem of Fáry states that every planar graph can be represented by an embedding in which every edge is represented by a straight line segment. We consider generalizations of Fáry's theorem to surfaces equipped with Riemannian metrics. In this setting, we require that every edge is drawn as a shortest path between its two endpoints and we call an embedding with this property a shortest path embedding. The main question addressed in this paper is whether given a closed surface S, there exists a Riemannian metric for which every topologically embeddable graph admits a shortest path embedding. This question is also motivated by various problems regarding crossing numbers on surfaces.

We observe that the round metrics on the sphere and the projective plane have this property. We provide flat metrics on the torus and the Klein bottle which also have this property.

Then we show that for the unit square flat metric on the Klein bottle there exists a graph without shortest path embeddings. We show, moreover, that for large g, there exist graphs G embeddable into the orientable surface of genus g, such that with large probability a random hyperbolic metric does not admit a shortest path embedding of G, where the probability measure is proportional to the Weil-Petersson volume on moduli space.

Finally, we construct a hyperbolic metric on every orientable surface S of genus g, such that every graph embeddable into S can be embedded so that every edge is a concatenation of at most O(g) shortest paths.

#### Daniel Gonçalves - 3-colorable planar graphs are 3-DIR

#### Louis Esperet - Box representations of embedded graphs

A box in R<sup>4</sup> is the cartesian product of d intervals of R. For instance, a box in R<sup>2</sup> is an axis-parallel rectangle. It was proved by Thomassen in 1986 that every planar graph can be represented as intersection graph of boxes in R<sup>3</sup> (i.e. each vertex corresponds to a box of R<sup>3</sup> and two vertices are adjacent if and only if their boxes intersect).

In this talk, I will explain how this result can be extended to graphs embedded in fixed surfaces. I will show that every graph of genus g can be represented as the intersection graph of boxes in dimension sqrt(g) log g, while there is a lower bound of order sqrt(g log g). I will also show that if a graph is embedded in a fixed surface and has no small noncontractible cycles, then it can be represented as the intersection graph of boxes in R^5.

#### Matej Stehlik - Edge- and face-width of projective quadrangulations

Graphs which embed in the projective plane so that all faces are bounded by four edges are called projective quadrangulations. They are a fascinating class of graphs with some surprising properties. In this talk, we will give upper bounds on the edge-width and the face-width of non-bipartite projective quadrangulations. The former is sharp, while the latter is a constant away from the optimal. We will also discuss some related questions.