Some Thoughts on Frederic's Mathematics and Mathematical Legacy

Bruce Reed

Grenoble, France

September 4th 2019

Drawing on Plato's Wall

2004 Isaiah Berlin Lecture by Tom Stoppard

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Artists are *divinely inspired maniacs* and *the unacknowledged legislators of the world.*

The Early Days

Thèse

Paris 1984

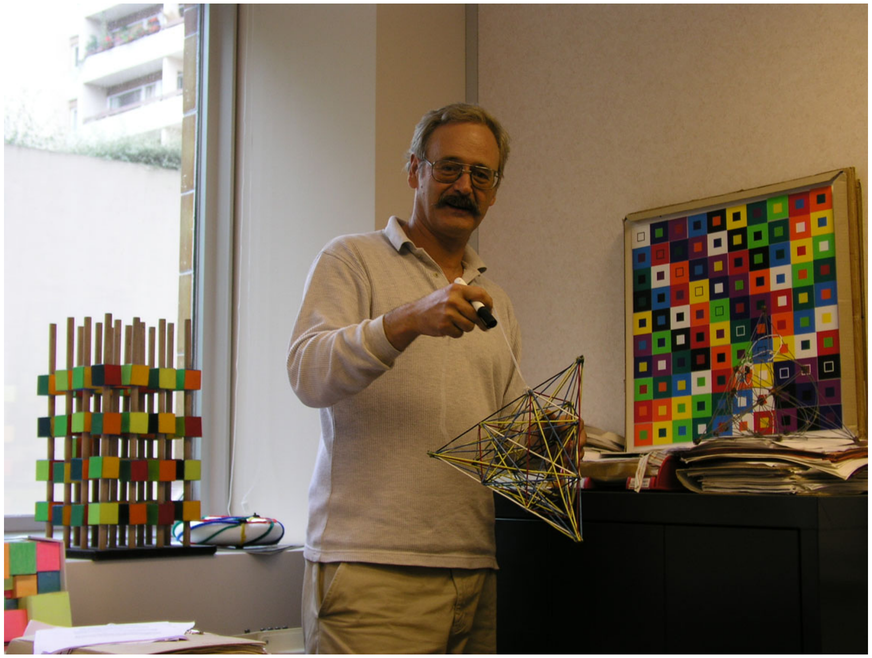
On The Existence Of Kernels in Perfect Graphs.

Supervisors: Berge and Duchet

Home

Work

Duchet



Meyniel

Frederic and Mostafa in 1984

Tours

Algeria

A Favourite Pastime

Solvable Graphs

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Conjecture(Berge and Duchet, 1983): A graph is solvable if and only if it is perfect.

Theorem(Maffray 1988): A line graph is solvable iff it is perfect.

Even Pairs

Europ. J. Combinatorics (1987) **8**, 313–316

A New Property of Critical Imperfect Graphs and some Consequences

H. MEYNIEL

We prove a new property of critical imperfect graphs. As a consequence, we define a new class of perfect graphs. This class contains perfectly orderable graphs and graphs in which that every odd cycle has two chords.

1. INTRODUCTION

A graph G is said to be γ-perfect if the chromatic number $\gamma(G')$ of any induced subgraph G' of G is equal to $\omega(G')$, the size of the largest complete subgraph in G'. G is said to be θ-perfect if and only if its complementary $\bar{G}$ is γ-perfect.

Lovasz [12] or [13] proved that a graph is γ-perfect if and only if it is θ-perfect. We can speak of perfect graphs instead of θ or γ-perfect graphs.

The well-known 'strong perfect graph conjecture' due to C. Berge (see for instance [2]), states that G is perfect if and only if G contains neither an odd hole (i.e. an odd cycle without a chord), nor an antihole (i.e. a hole in $\bar{G}$). For further information on the strong perfect graph conjecture the reader is referred to [2].

In the present paper we give a new property of critical imperfect graphs and as a consequence, we define a new class of perfect graphs. this new class contains two known classes of perfect graphs: perfectly orderable graphs defined by V. Chvatal [6] and graphs such that every odd cycle has two chords [14].

2. DEFINITIONS AND NOTATIONS

Definitions and notations are classical, see [1]. We consider here only finite simple undirected graphs unless otherwise specified. An *induced path* between two vertices x and y is a path $(x_0 = x, x_1, \ldots, x_m = y)$ such that (x_i, x_j), $i < j$, is an edge of G if and only if $j = i + 1$, $i = 0, 1, \ldots, m - 1$. An induced path is *even* if it contains an even number of edges. The *length* of a path is its number of edges.

A graph G is said to be *critical imperfect* if any proper induced subgraph of G is perfect and if G itself is not perfect.

The size of the complete largest subgraph in G is denoted by $\omega(G)$, the size of the largest independent subset by $\alpha(G)$, the chromatic number of G by $\gamma(G)$. The set of vertices of G adjacent to a vertex x is denoted by $\Gamma_G(x)$. If G is directed, we denote by $\Gamma_G^+(x)$, (resp. $\Gamma_G^-(x)$), the set of vertices y such that (x, y), (resp. (y, x)), is an arc of G.

3. THE RESULTS

We shall prove that a critical imperfect graph is such that each pair of different vertices is joined by an odd induced path. In order to prove this result, we need two lemmas.

LEMMA 1. *Let G be a graph. If two non adjacent vertices x and y of G are not linked by an induced path of length* 3, *the graph G' obtained from G by identifying x and y satisfies* $\omega(G') = \omega(G)$.

PROOF. Clearly $\omega(G') \geqslant \omega(G)$. Suppose we have $\omega(G') > \omega(G)$, then there must exist in G some complete subgraph on a set of vertices K of size $\omega(G)$ such that

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Fonlupt+Uhry 1980

No minimal imperfect graph has an even pair Meyniel 1984

If no odd cycle of G has <two chords then G has an even pair. Me 1984

A New Strand

Star Cutsets, Skew Cutsets, and WT Graphs

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Thm: For every weakly triangulated G with |V(G)|>3, either G or $\bar{G}$ has a star cutset.

Hayward 85.

A Conjecture

For every Berge graph either:

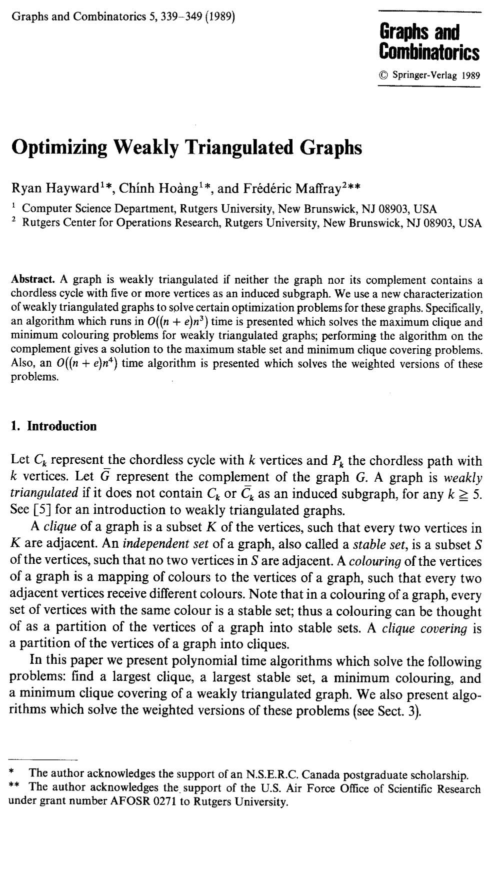
G or $\bar{G}$ contains a star(skew) cutset,

G or $\bar{G}$ contains an even pair, or

G or $\bar{G}$ is LGBG.

Ph.D.
Rutgers, 1989
Structural Aspects of Perfect Graphs
Supervisor: P. Hammer.

Two Pairs



Graphs and Combinatorics 5, 339–349 (1989)

Graphs and Combinatorics

Optimizing Weakly Triangulated Graphs

Ryan Hayward[1]*, Chính Hoàng[1]*, and Frédéric Maffray[2]**

[1] Computer Science Department, Rutgers University, New Brunswick, NJ 08903, USA

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Abstract. A graph is weakly triangulated if neither the graph nor its complement contains a chordless cycle with five or more vertices as an induced subgraph. We use a new characterization of weakly triangulated graphs to solve certain optimization problems for these graphs. Specifically, an algorithm which runs in $O((n + e)n^3)$ time is presented which solves the maximum clique and minimum colouring problems for weakly triangulated graphs; performing the algorithm on the complement gives a solution to the maximum stable set and minimum clique covering problems. Also, an $O((n + e)n^4)$ time algorithm is presented which solves the weighted versions of these problems.

1. Introduction

Let C_k represent the chordless cycle with k vertices and P_k the chordless path with k vertices. Let $\bar{G}$ represent the complement of the graph G. A graph is *weakly triangulated* if it does not contain C_k or $\bar{C}_k$ as an induced subgraph, for any $k \geq 5$. See [5] for an introduction to weakly triangulated graphs.

A *clique* of a graph is a subset K of the vertices, such that every two vertices in K are adjacent. An *independent set* of a graph, also called a *stable set*, is a subset S of the vertices, such that no two vertices in S are adjacent. A *colouring* of the vertices of a graph is a mapping of colours to the vertices of a graph, such that every two adjacent vertices receive different colours. Note that in a colouring of a graph, every set of vertices with the same colour is a stable set; thus a colouring can be thought of as a partition of the vertices of a graph into stable sets. A *clique covering* is a partition of the vertices of a graph into cliques.

In this paper we present polynomial time algorithms which solve the following problems: find a largest clique, a largest stable set, a minimum colouring, and a minimum clique covering of a weakly triangulated graph. We also present algorithms which solve the weighted versions of these problems (see Sect. 3).

* The author acknowledges the support of an N.S.E.R.C. Canada postgraduate scholarship.
** The author acknowledges the support of the U.S. Air Force Office of Scientific Research under grant number AFOSR 0271 to Rutgers University.

x and y form a 2-pair if and only if they are in the same component of G-($N(x) \cap N(y)$).

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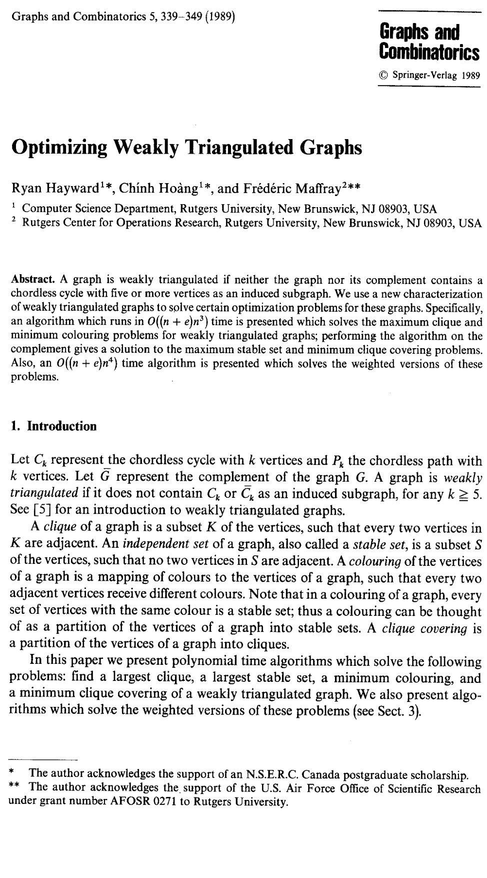

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Theorem: Every WT- graph which is not a clique contains a 2-pair.

Hayward,Hoang, Maffray '87

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Theorem: Every WT- graph which is not a clique contains a 2-pair.

Hayward,Hoang, Maffray '87

Observation: Contracting a 2-pair in a WT graph yields a WT graph.

Completely Separable Graphs

Discrete Applied Mathematics 27 (1990) 85-99 85
North-Holland

COMPLETELY SEPARABLE GRAPHS*

Peter L. HAMMER and Frédéric MAFFRAY**
RUTCOR, Rutgers Center for Operations Research, Hill Center, Busch Campus, Rutgers University, New Brunswick, NJ 08903, USA

Received 30 May 1989

We define a property of Boolean functions called separability, and specialize it for a class of functions naturally associated with graphs. "Completely separable graphs" are then derived and characterized in particular by the existence of two crossing chords in any cycle of length at least five. This implies that completely separable graphs are perfect. We present linear time algorithms for the recognition and for the usual optimization problems (maximum weighted stable set and maximum weighted clique).

1. Introduction

Many problems that arise in combinatorial optimization can be expressed as:

$$\text{Maximize } w \cdot x, \quad \text{subject to } f(x)=0, \tag{1}$$

where f is a given Boolean function of the n-dimensional vector x. For example, the weighted stability number of a graph, in which the vertices $x_1, \dots, x_n$ are assigned weights $w_1, \dots, w_n$, is given by any solution of problem (1), when f is a certain Boolean function associated with the graph. Similarly, the maximum weight matching problem can be formulated as an instance of (1).

In the general case, problem (1) is known to be NP-complete. One idea to help solving such a problem is to try to reduce it to smaller (and easier) instances.

Definition 1.1. We will say that a Boolean function f on n variables $x_1, \dots, x_n$ is *separable* if we can write

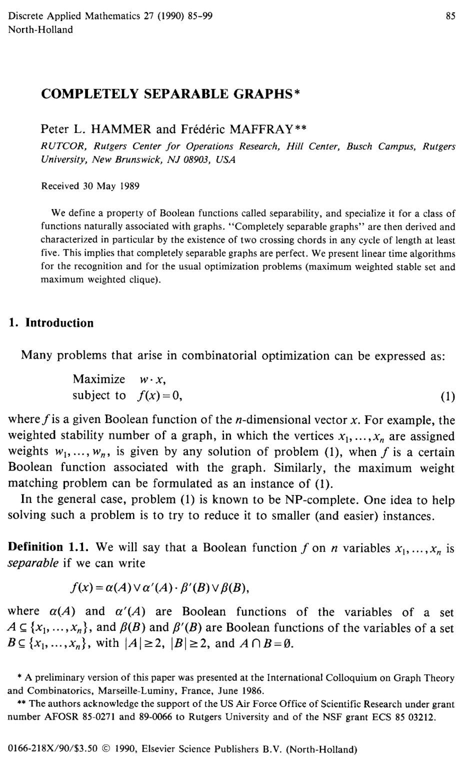
$$f(x)=\alpha(A) \vee \alpha'(A) \cdot \beta'(B) \vee \beta(B),$$

where $\alpha(A)$ and $\alpha'(A)$ are Boolean functions of the variables of a set $A \subseteq \{x_1, \dots, x_n\}$, and $\beta(B)$ and $\beta'(B)$ are Boolean functions of the variables of a set $B \subseteq \{x_1, \dots, x_n\}$, with $|A| \geq 2$, $|B| \geq 2$, and $A \cap B = \emptyset$.

* A preliminary version of this paper was presented at the International Colloquium on Graph Theory and Combinatorics, Marseille-Luminy, France, June 1986.
** The authors acknowledge the support of the US Air Force Office of Scientific Research under grant number AFOSR 85-0271 and 89-0066 to Rutgers University and of the NSF grant ECS 85 03212.

G is *completely separable* if every cycle of G of length at least 5 has at least two chords.

Completely Separable Graphs



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COMPLETELY SEPARABLE GRAPHS*

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We define a property of Boolean functions called separability, and specialize it for a class of functions naturally associated with graphs. "Completely separable graphs" are then derived and characterized in particular by the existence of two crossing chords in any cycle of length at least five. This implies that completely separable graphs are perfect. We present linear time algorithms for the recognition and for the usual optimization problems (maximum weighted stable set and maximum weighted clique).

1. Introduction

Many problems that arise in combinatorial optimization can be expressed as:

$$\text{Maximize} \quad w \cdot x, \qquad \text{subject to} \quad f(x) = 0, \qquad (1)$$

where f is a given Boolean function of the n-dimensional vector x. For example, the weighted stability number of a graph, in which the vertices $x_1, \dots, x_n$ are assigned weights $w_1, \dots, w_n$, is given by any solution of problem (1), when f is a certain Boolean function associated with the graph. Similarly, the maximum weight matching problem can be formulated as an instance of (1).

In the general case, problem (1) is known to be NP-complete. One idea to help solving such a problem is to try to reduce it to smaller (and easier) instances.

Definition 1.1. We will say that a Boolean function f on n variables $x_1, \dots, x_n$ is *separable* if we can write

$$f(x) = \alpha(A) \vee \alpha'(A) \cdot \beta'(B) \vee \beta(B),$$

where $\alpha(A)$ and $\alpha'(A)$ are Boolean functions of the variables of a set $A \subseteq \{x_1, \dots, x_n\}$, and $\beta(B)$ and $\beta'(B)$ are Boolean functions of the variables of a set $B \subseteq \{x_1, \dots, x_n\}$, with $|A| \geq 2$, $|B| \geq 2$, and $A \cap B = \emptyset$.

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G is *completely separable* if every cycle of G of length at least 5 has at least two chords.

Theorem: (a) G is completely separable if and only if for every pair u and v of vertices of G, all the paths between u and v have the same length,

(b) G is completely separable iff. Every induced subgraph has a pair of twins or a vertex of degree 1.

Hammer and Maffray 1987

Later Accomplishments

More on Kernels

Kernels and Choosability

List Colouring Conjecture: For any line graph G, the chromatic number and choice number of G are equal Vizing 1976, ERT 1979.

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Galvin's Theorem: LCC for bipartite G. 1994

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Proof of Galvin's Theorem:

Lemma 1: If G has an orientation such that every induced subgraph has a kernel then its choice number exceeds the maximum outdegree by at most 1 BBS '92

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Lemma 3: Every LGBG G has a normal orientation of outdegree at most $\chi(G)$-1.

Some Related Papers by Frederic

On kernels in perfect graphs Blidia, Duchet,Meyniel 1993

On the orientation of Meyniel graphs. BDM 1994

Graphs whose choice number is equal to their chromatic numbers Gravier and Maffray 1998

Some operations preserving the existence of kernels BDJMM 1999

On a list colouring problem Gravier Maffray Mohar 2003

On the choice number of claw-free perfect graphs GM 2004

Precolouring Extension of Co-Meyniel Graphs JLM 2007

List colouring claw free graphs with small clique number EGM 2014

On the choosability of claw free perfect graphs GMP 2016

Frederic’s papers on domination

On lower independence and domination numbers in graphs Blidia, Chellali, Maffray 2005.

Exact double domination in graphs Chellali, Khelladi, Maffray 2005

Extremal graphs for a new upper bound on domination parameters in graphs BCM 2006.

Small step-dominating sets in tree Maffray, Rautenbach 2007.

Locating domination & identifying codes in trees BCM, Moncel, Sefri 2007

Extremal perfect graphs for a bound on the domination number BCM 2008

Double Domination edge removal critical graphs Khelifi, B, C, M 2010

Connected domination dot-critical graphs C, M, Tablennehas 2010

Characterization of trees with unique minimum location-dominating sets B, C, Lounes, M 2011

Ceci N'est Pas Une Session Algérienne

Frederic and Mustafa, Grenoble 1993

Frederic at Blidia 2006 (I of II)

Frederic at Blidia 2006 (II of II)

Frederic at Blidia 2009 (I of II)

Frederic at Blidia 2009 (II of II)

More On The Structure Of Perfect Graphs

Frederic's Papers on
Even Pairs And Perfectly Contractile Graphs

Frederic's Papers on Even Pairs And Perfectly Contractile Graphs

Opposition Graphs are Quasi-Parity Graphs Hoang+M 1989

On Slim Graphs, even pairs, and star cutsets Hoang +Maffray 1992

Colouring, Path Parity, and Perfection EdFLMPR 1997

On Planar Perfectly Contractile Graphs LMR 1997.

Even Pairs in Claw-free Perfect Graphs, Linhares-Sales,M 1998

Recognizing Planar Strict Quasi-Parity Graphs LMR 2001

Even Pairs EdFLMPR 2001

Even Pairs in Square-Free Berge Graphs LM 2003

On Dart-free Perfectly Contractile Graphs LM 2004

Algorithms for Perfectly Contractile Graphs

Maffray and Trotignon 2005

A class of Perfectly Contractile Graphs MT 2006

Even Pairs in Bull-Reducible Graphs dFMM 2006

Colouring Bull-free Perfectly Contractile Graphs

Levesque & Maffray 2008

On Planar Quasi-Parity Graphs LMR 2008

Colouring Artemis Graphs LMRT 2009

Even Pairs in Square-Free Berge Graphs M 2015

The Structure of
Claw-Free (Perfect) Graphs

Elementary and Peculiar Graphs

A graph is *elementary* if we can two colour its edge so that no induced path of kength three is monochromatic.

A graph is *peculiar* if it can be partitioned into 3 cliques, contains no cycle of length exceeding four, and satisfies certain additional properties.

Theorem; Every claw-free perfect graph is elementary or peculiar.

Chvatal and Sbihi 1988

The Structure of Elementary Graphs

An edge of G is *flat* if it is in no triangle.

We *augment* G on a matching M by substituting a clique K_x for each vertex x of M and then deleting some of the edges between K_x and K_y for every

Theorem: Every elementary graph arises by augmenting a matching of flat edges in an LGBG.

Maffray and Reed 1999

The Structure of Quasi-Line Graphs Without Homogenous Pairs of Cliques.

A graph is *quasi-line* if the neighbourhood of every vertex can be partititoned into two cliques.

A linear (circular) interval graph is the intersection graph of a set of intervals on the real line (on a circle).

An ordered linear strip consists of a linear interval graph G, a clique consisiting of some set of vertices whose interval starts first and a clique consisting of some set of vertices whose interval starts last.

A pair of (disjoint) cliques is *homogenous* if every vertex outside the pair sees all or none of both cliques.

An *augmentation* of the line graph of H is obtained from a set of ordered linear strips indexed by the arcs of an orientation of H by adding edges between all the cliques corresponding to the edges incident to v for all $v \epsilon V(H)$

Theorem: Every connected quasi line graph with no homogeneous pair of cliques is either a circular interval graph or the augmentation of a line graph.

The Structure of Claw Free Graphs

An ordered strip consists of a graph G and ordered pair of cliques of G.

A 1-join of G_1 and G_2 is obtained by deleting a simplicial vertex a_i of G_i and a simplicial vertex a_2 of G_2 and

adding all edges frm $N(a_1)$ to $N(a_2)$

If G has a 1-join it has a star cutset.

An *augmentation* of the line graph of H is obtained from a set of ordered strips indexed by the arcs of an orientation of H by adding edges between all the cliques corresponding to the edges incident to v for all $v \epsilon V(H)$

Theorem: Every connected claw-free graph with stability number at least four which does not arise by a 1-join and contains no homogeneous pair of cliques is either a circular interval graph or the augmentation of a line graph using special types of strips.

Chudnovsky and Seymoure 2005

An Open Question

Is $\chi(G) \leq \frac{6}{5}\chi_f(G)$ for every claw-free G?

Thank you for your attention!