LP based approximation of induced matchings

Dieter Rautenbach

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Joint with Julien Baste and Maximilian Fürst

Joint work with Frédéric

- F. Maffray and D. Rautenbach,
  Small Step-Dominating Sets in Trees,
  Discrete Math. **307** (2007), 1212-1215.

- S. Chaplick, M. Fürst, F. Maffray, and D. Rautenbach,
  On some Graphs with a Unique Perfect Matching,
  Inf. Process. Lett. **139** (2018), 60-63.

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Observation (Zito '99)

If G is a Δ-regular graph, then

$$\nu_s(G) \leq \frac{m(G)}{2\Delta - 1}.$$

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There is an efficient algorithm that, for a given graph G of maximum degree at most Δ at least 3, produces an induced matching M in G with

$$|M| \geq \frac{m(G)}{1.5\Delta^2 - 0.5\Delta}.$$

ILP for $\nu_s(G)$

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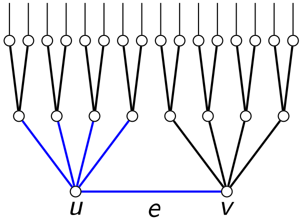

- For $u \in V(G)$: $\delta_G(u)$

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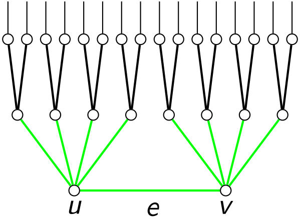

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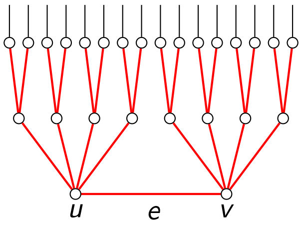

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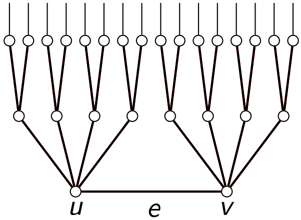

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s.t. & \sum_{f \in \delta_G(e)} x_f \le 1 & & \forall e \in E(G) \\
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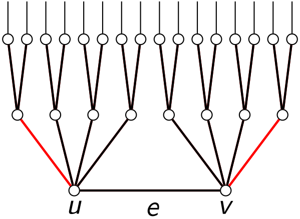

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Conjecture (Baste, Fürst, & R '18+)

If G is a graph with maximum degree at most Δ, then

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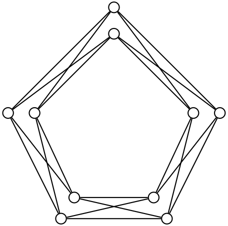

- $\nu_s(G) = 1$
- $m(G) = 1.25\Delta^2$ (for even Δ)
- $\nu_s^*(G) = m(G)/(2\Delta - 1)$ (for even Δ)

ILP for $\nu_s(G)$

Theorem (Baste, Fürst, & R '18+)

If G is as above and each component has order at least 3, then

$$\nu_s^*(G) \leq \frac{\Delta}{2\Delta+1} n(G)$$

with equality if and only if each component of G is a complete subdivision of $K_{1,\Delta}$.

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Combining this with $\nu_s(G) \geq n(G)/6$ (Joos, Sasse, R '14) for connected subcubic graphs G of order at least 7, yields an approximation algorithm with factor

$$\frac{18}{7} \approx 2.57$$

for subcubic graphs.

LP-based approximation

Theorem (Baste, Fürst, & R '18+)

There is an efficient algorithm that, for a given subcubic graph G, produces an induced matching M in G as well as a feasible solution $(y_e)_{e\in E(G)}$ of (D) with

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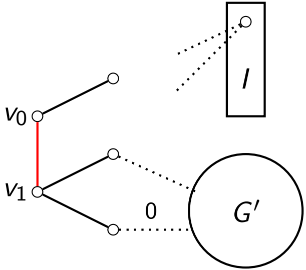

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- $y\left(E(H)\right) \leq \frac{7}{3}$.

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Conjecture (Erdős & Nešetřil '89)

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- $\chi'_s(G) \leq 1.998\Delta^2$ for large Δ (Molloy & Reed '97)

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- $\chi'_s(G) \leq 1.835\Delta^2$ for large Δ (Bonamy et al. '18+)

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Theorem (Baste, Fürst, & R '18+)

There is an efficient algorithm that, for a given graph G of maximum degree at most $\Delta \geq 3$, produces an induced matching M in G with

$$|M| \geq \frac{\nu_s^*(G)}{(1-\epsilon)\Delta + 0.5}, \text{ where } \epsilon \approx 0.02005.$$

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Let $\epsilon \approx 0.02005$ *and* $f = (1-\epsilon)\Delta + 0.5$.

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that is, the lemma follows for a suitable β.

LP-based approximation

(ϵ, c) is an optimal solution of the following quadratic program:

$$(Q)\left\{\begin{array}{lrcl} \max & \epsilon & & \\ s.th. & 1.5\left(1+\frac{\epsilon(2c-1+\epsilon)}{1-c-\epsilon}\right) & \leq & 2c(1-\epsilon) \\ & \epsilon & \leq & (1-c)^2 \\ & \epsilon + c & < & 1 \\ & \epsilon, c & > & 0 \end{array}\right.$$

Standard software yields

$$\epsilon \approx 0.02005 \qquad \text{and} \qquad c \approx 0.85838,$$

and feasibility suffices for the proof.

Thank you!