My publications with Frédéric Maffray

- V. Campos, C. Linhares Sales, F. Maffray, A. Silva. **b-chromatic number of cacti**. LAGOS 2009.
- V. Campos, A. Gyárfás, F. Havet, C. Linhares Sales, F. Maffray. **New bounds on the Grundy number of products of graphs**. Journal of Graph Theory 2012.

Adapting the Directed Grid Theorem into an FPT Algorithm

V. Campos [1] R. Lopes [1] A. K. Maia [1] I. Sau [2]

[1]ParGO Group, Universidade Federal do Ceará, Brazil
[2]CNRS, LIRMM, Université de Montpellier, France

A Tribute to Frédéric Maffray, Grenoble
September, 2019

XP and FPT

Problem $\mathcal{P}$ of size n with a parameter k:

- $\mathcal{P} \in XP \implies \mathcal{P}$ can be solved in $O\left(n^{f(k)}\right)$.

Examples

XP and FPT

Problem $\mathcal{P}$ of size n with a parameter k:

- $\mathcal{P} \in XP \implies \mathcal{P}$ can be solved in $O\left(n^{f(k)}\right)$.

Examples

$$O\left(n^{2k}\right)$$

XP and FPT

Problem $\mathcal{P}$ of size n with a parameter k:

- $\mathcal{P} \in XP \implies \mathcal{P}$ can be solved in $O\left(n^{f(k)}\right)$.

Examples

$$O\left(n^{2k}\right)$$

Poly time for fixed k.

XP and FPT

Problem $\mathcal{P}$ of size n with a parameter k:

- $\mathcal{P} \in XP \implies \mathcal{P}$ can be solved in $O\left(n^{f(k)}\right)$.
- $\mathcal{P} \in FPT \implies \mathcal{P}$ can be solved in $O\left(f(k) \cdot n^c\right)$.

Examples

$$O\left(n^{2k}\right)$$

Poly time for fixed k.

XP and FPT

Problem $\mathcal{P}$ of size n with a parameter k:

- $\mathcal{P} \in XP \implies \mathcal{P}$ can be solved in $O\left(n^{f(k)}\right)$.
- $\mathcal{P} \in FPT \implies \mathcal{P}$ can be solved in $O\left(f(k) \cdot n^c\right)$.

Examples

$$O\left(n^{2k}\right) \qquad O\left(2^k \cdot n^2\right)$$

Poly time for fixed k.

XP and FPT

Problem $\mathcal{P}$ of size n with a parameter k:

- $\mathcal{P} \in XP \implies \mathcal{P}$ can be solved in $O\left(n^{f(k)}\right)$.
- $\mathcal{P} \in FPT \implies \mathcal{P}$ can be solved in $O\left(f(k) \cdot n^c\right)$.

Examples

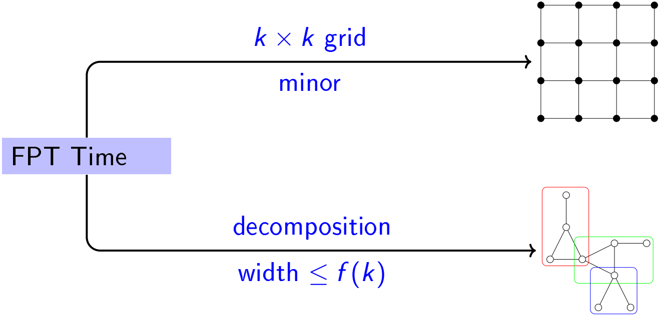
$$O\left(n^{2k}\right)$$

Poly time for fixed k.

$$O\left(2^k \cdot n^2\right)$$

Poly exponent independent of k.

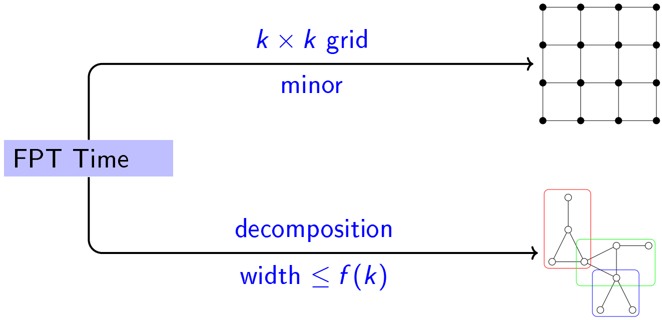
(Undirected) Grid Theorem



Grid Theorem

N. Robertson and P. Seymour.
Graph minors V. Excluding a planar graph.
Journal of Combinatorial Theory, Series B, 1986.

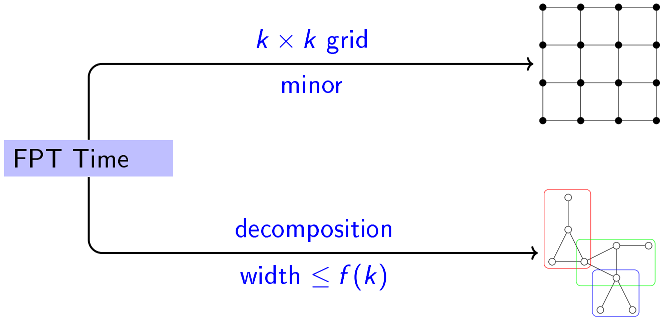
(Undirected) Grid Theorem



Applications

- Key ingredient in proof of Wagner's Conjecture.

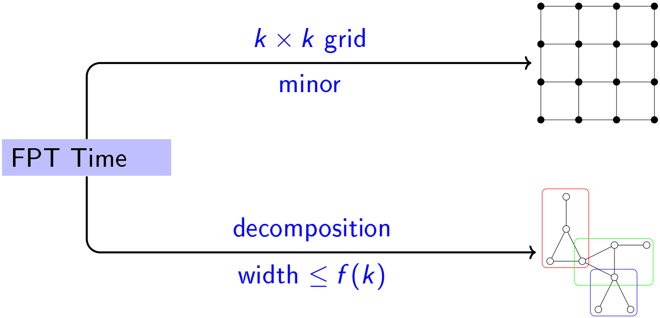
(Undirected) Grid Theorem



Applications

- Key ingredient in proof of Wagner's Conjecture.
- Base of Bidimensionality Theory

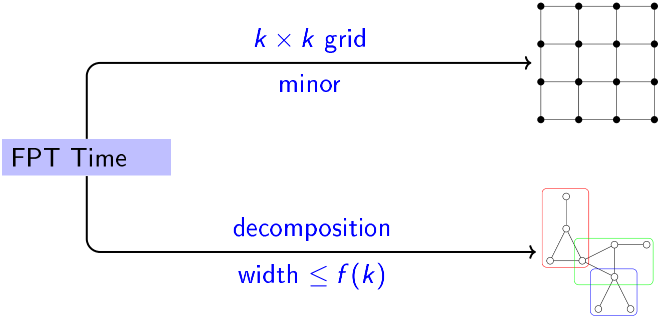
(Undirected) Grid Theorem



Applications

- Key ingredient in proof of Wagner's Conjecture.
- Base of Bidimensionality Theory
- Basis for several other structure theorems

(Undirected) Grid Theorem



Conjecture: Directed version

Conjectured independently by

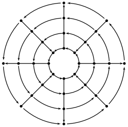
- Reed (1999)
- Johnson, Robertson, Seymour and Thomas (2001)

Directed Grid Theorem

XP Time

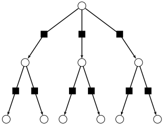
order k

cylindrical grid minor



decomposition

width $\leq f(k)$



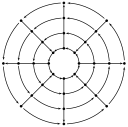
Proof - 20 years later

K. Kawarabayashi and S. Kreutzer.
The Directed Grid Theorem
STOC'15

Directed Grid Theorem

order k

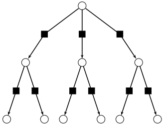
cylindrical grid minor



XP Time

decomposition

width $\leq f(k)$



Our result

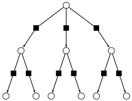
$$\text{XP} \to \text{FPT}$$

Understanding the Directed Grid Theorem

XP Time

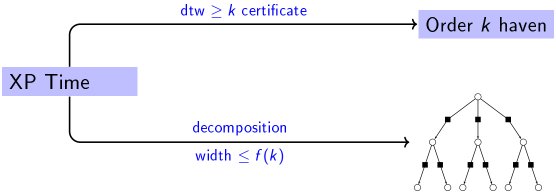
dtw $\geq k$ certificate $\rightarrow$ Order k haven

decomposition width $\leq f(k)$ $\rightarrow$



T. Johnson, N. Robertson, P. Seymour and R. Thomas.
Directed tree-width
J. Comb. Theory, Ser. B, 2001

Understanding the Directed Grid Theorem



- Big haven $\Rightarrow$ Big bramble. (Existential)

B. Reed.
Introducing directed tree-width
ENDM, 1999

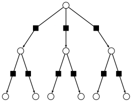
Kawarabayashi and Kreutzer.
The Directed Grid Theorem
STOC'15

Understanding the Directed Grid Theorem

XP Time

dtw $\geq k$ certificate → Order k haven

decomposition width $\leq f(k)$ →

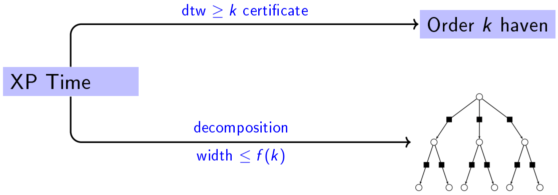


- Big haven $\Rightarrow$ Big bramble. ~~(Existential)~~

B. Reed.
Tree width and tangles, (...)
Surveys in Combinatorics, 1997

Kawarabayashi and Kreutzer.
The Directed Grid Theorem
STOC'15

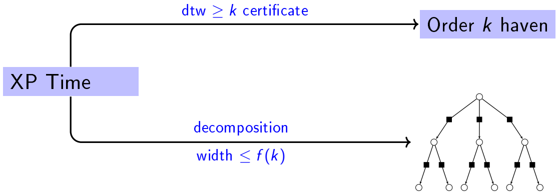
Understanding the Directed Grid Theorem



- Big haven $\Rightarrow$ Big bramble.
- Big bramble $\Rightarrow$ Well linked long path. (High cost)

K. Kawarabayashi and S. Kreutzer.
The Directed Grid Theorem
STOC'15

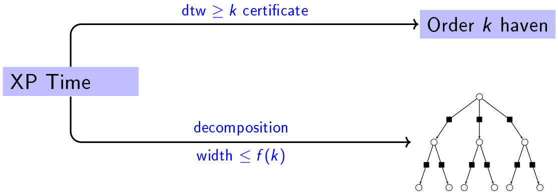
Understanding the Directed Grid Theorem



- Big haven $\Rightarrow$ Big bramble.
- Big bramble $\Rightarrow$ Well linked long path. (High cost)
- Well linked long path $\Rightarrow$ Cylindrical Grid. (FPT)

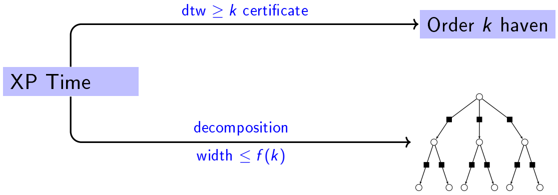
K. Kawarabayashi and S. Kreutzer.
The Directed Grid Theorem
STOC'15

Understanding the Directed Grid Theorem



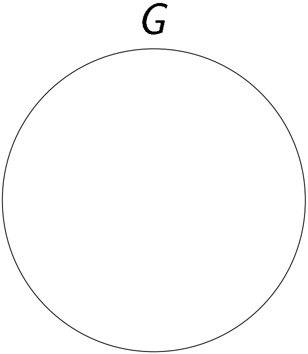
- Big haven $\Rightarrow$ Big bramble.
- Big bramble $\Rightarrow$ Well linked long path. (High cost)
- Well linked long path $\Rightarrow$ Cylindrical Grid. (FPT)
- Objective 1: FPT haven algorithm

Understanding the Directed Grid Theorem

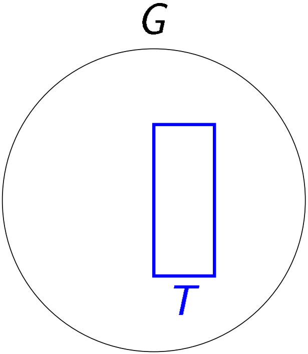


- Big haven $\Rightarrow$ Big bramble.
- Big bramble $\Rightarrow$ Well linked long path. (High cost)
- Well linked long path $\Rightarrow$ Cylindrical Grid. (FPT)
- Objective 1: FPT haven algorithm
- Objective 2: Handle brambles and build well linked path

Problem $\mathcal{P}$: XP part of JRST haven algorithm

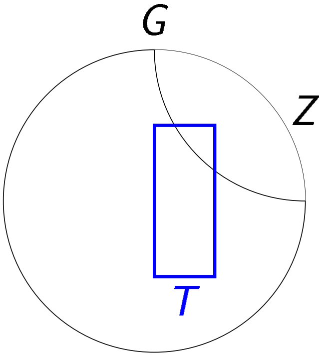


Problem $\mathcal{P}$: XP part of JRST haven algorithm



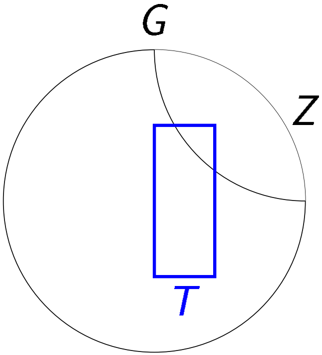
- Given $|T| \leq 2k - 1$.

Problem $\mathcal{P}$: XP part of JRST haven algorithm



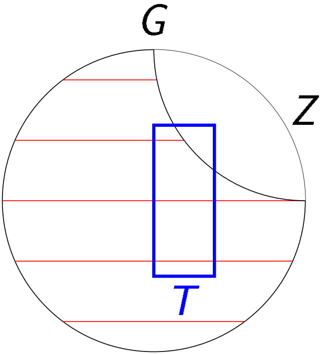
- Given $|T| \leq 2k - 1$.
- Find $Z \subseteq V(G)$:

Problem $\mathcal{P}$: XP part of JRST haven algorithm



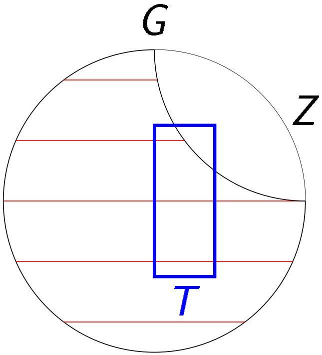
- Given $|T| \leq 2k - 1$.
- Find $Z \subseteq V(G)$:
  - $|Z| \leq k - 1$; and

Problem $\mathcal{P}$: XP part of JRST haven algorithm



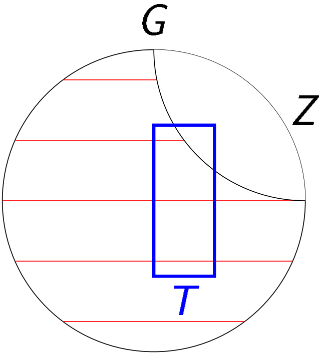
- Given $|T| \leq 2k - 1$.
- Find $Z \subseteq V(G)$:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong components of $G \setminus Z$.

Problem $\mathcal{P}$: XP part of JRST haven algorithm



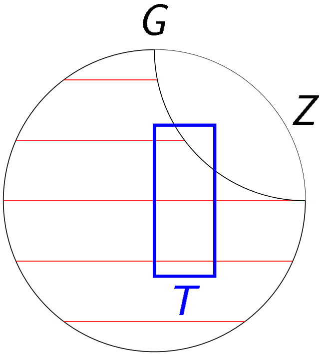
- Given $|T| \leq 2k-1$.
- Find $Z \subseteq V(G)$:
  - $|Z| \leq k-1$; and
  - $|C \cap T| \leq k-1$ for strong components of $G \setminus Z$.
- Enumerate all $\binom{n}{k-1}$ subsets of $V(G)$.

Problem $\mathcal{P}$: XP part of JRST haven algorithm



- Given $|T| \leq 2k - 1$.
- Find $Z \subseteq V(G)$:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong components of $G \setminus Z$.
- Enumerate all $\binom{n}{k-1}$ subsets of $V(G)$.
- $O(n^k)$

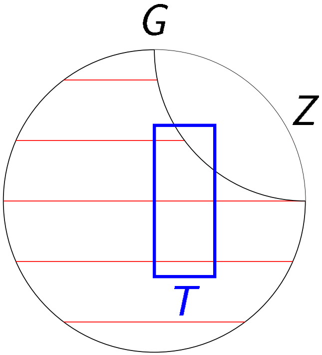
Problem $\mathcal{P}$: XP part of JRST haven algorithm



- **Any negative $\Rightarrow$ haven**

- Given $|T| \leq 2k-1$.
- Find $Z \subseteq V(G)$:
  - $|Z| \leq k-1$; and
  - $|C \cap T| \leq k-1$ for strong components of $G \setminus Z$.
- Enumerate all $\binom{n}{k-1}$ subsets of $V(G)$.
- $O(n^k)$

Problem $\mathcal{P}$: XP part of JRST haven algorithm



- **Any negative $\Rightarrow$ haven**
- **All positive $\Rightarrow$ decomposition**

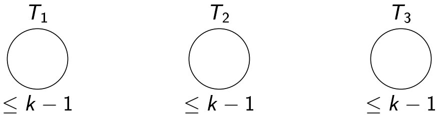
- Given $|T| \leq 2k - 1$.
- Find $Z \subseteq V(G)$:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong components of $G \setminus Z$.
- Enumerate all $\binom{n}{k-1}$ subsets of $V(G)$.
- $O(n^k)$

Similar problem $\mathcal{P}'$

- $|T| \leq 2k - 1.$

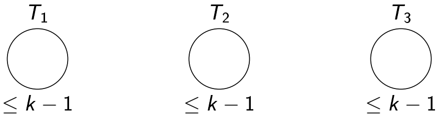
Similar problem $\mathcal{P}'$

- $|T| \leq 2k - 1$.
- Partition $T_1, \ldots, T_r$ of T with $|T_i| \leq k - 1$.



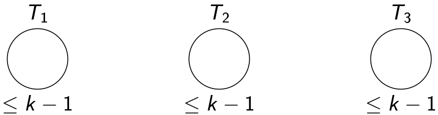
Similar problem $\mathcal{P}'$

- $|T| \leq 2k - 1$.
- Partition $T_1, \ldots, T_r$ of T with $|T_i| \leq k - 1$.
- find $Z \subseteq V(G)$ such that:



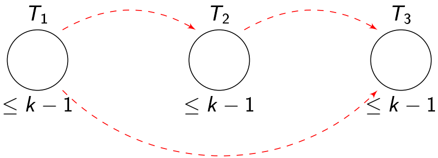
Similar problem $\mathcal{P}'$

- $|T| \leq 2k - 1$.
- Partition $T_1, \ldots, T_r$ of T with $|T_i| \leq k - 1$.
- find $Z \subseteq V(G)$ such that:
  1. $|Z| \leq k - 1$; and



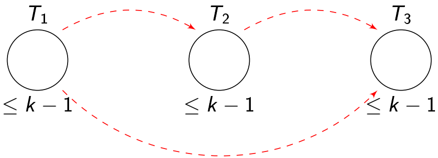
Similar problem $\mathcal{P}'$

- $|T| \leq 2k - 1$.
- Partition $T_1, \ldots, T_r$ of T with $|T_i| \leq k - 1$.
- find $Z \subseteq V(G)$ such that:
  1. $|Z| \leq k - 1$; and
  2. $G \setminus Z$ has no T_i to T_j path for $i < j$



Similar problem $\mathcal{P}'$

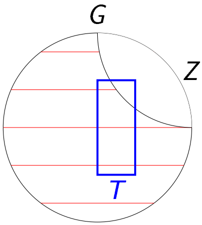
- $|T| \leq 2k - 1$.
- Partition $T_1, \ldots, T_r$ of T with $|T_i| \leq k - 1$.
- find $Z \subseteq V(G)$ such that:
  1. $|Z| \leq k - 1$; and
  2. $G \setminus Z$ has no T_i to T_j path for $i < j$

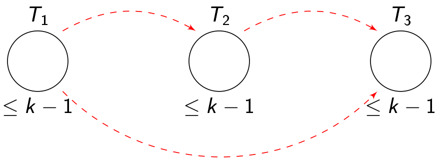


Reduces to

R. Erbacher, T. Jaeger, N. Talele and J. Teutsch
Directed Multicut with Linearly Ordered Terminals
CoRR abs/1407.7498, 2014

$\mathcal{P}$ and $\mathcal{P}'$

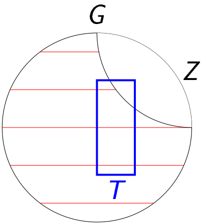


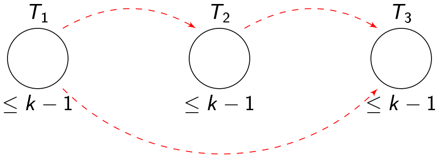


Lemma

$\mathcal{P}$ positive $\Leftrightarrow$ $\mathcal{P}'$ positive for some partition $T_1, \ldots, T_r$ of $T \setminus Z$

$\mathcal{P}$ and $\mathcal{P}'$

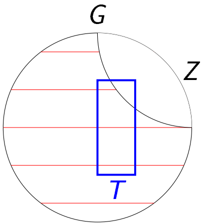


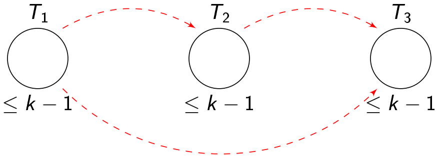


Lemma

$\mathcal{P}$ positive $\Leftrightarrow$ $\mathcal{P}'$ positive for some partition $T_1, \ldots, T_r$ of $T \setminus Z$

- Partitions of T is $f(k)$ ($|T| \leq 2k-1$)



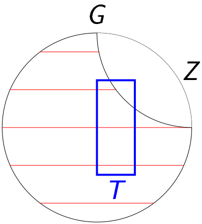


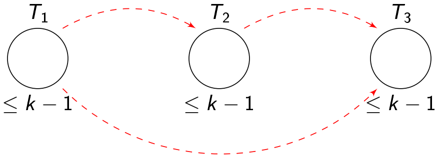
Lemma

$\mathcal{P}$ positive $\Leftrightarrow$ $\mathcal{P}'$ positive for some partition $T_1, \ldots, T_r$ of $T \setminus Z$

- Partitions of T is $f(k)$ ($|T| \leq 2k-1$)
- $\mathcal{P}$ is FPT

$\mathcal{P}$ and $\mathcal{P}'$





Lemma

$\mathcal{P}$ positive $\Leftrightarrow$ $\mathcal{P}'$ positive for some partition $T_1, \ldots, T_r$ of $T \setminus Z$

- Partitions of T is $f(k)$ ($|T| \leq 2k-1$)
- $\mathcal{P}$ is FPT
- FPT Haven Algorithm (Objective 1)

Understanding brambles

Definition (Brambles on digraphs)

- Family of strongly connected subgraphs $\mathcal{B} = \{B_1, \ldots, B_\ell\}$

Understanding brambles

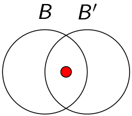
Definition (Brambles on digraphs)

- Family of strongly connected subgraphs $\mathcal{B} = \{B_1, \ldots, B_\ell\}$
- if $\{B, B'\} \subseteq \mathcal{B}$ then either

Understanding brambles

Definition (Brambles on digraphs)

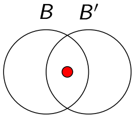
- Family of strongly connected subgraphs $\mathcal{B} = \{B_1, \ldots, B_\ell\}$
- if $\{B, B'\} \subseteq \mathcal{B}$ then either
    1. $V(B) \cap V(B') \neq \emptyset$ or

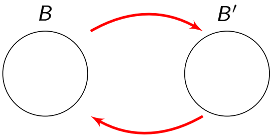


Understanding brambles

Definition (Brambles on digraphs)

- Family of strongly connected subgraphs $\mathcal{B} = \{B_1, \ldots, B_\ell\}$
- if $\{B, B'\} \subseteq \mathcal{B}$ then either
  1. $V(B) \cap V(B') \neq \emptyset$ or
  2. edges from B to B' and B' to B.

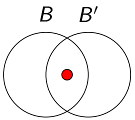


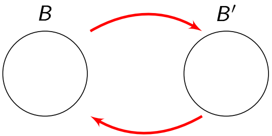


Understanding brambles

Definition (Brambles on digraphs)

- Family of strongly connected subgraphs $\mathcal{B} = \{B_1, \ldots, B_\ell\}$
- if $\{B, B'\} \subseteq \mathcal{B}$ then either
  1. $V(B) \cap V(B') \neq \emptyset$ or
  2. edges from B to B' and B' to B.



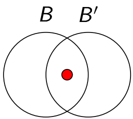


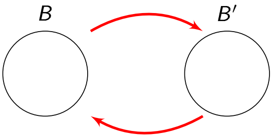
- **hitting set** of $\mathcal{B}$ = set of vertices touching every $B \in \mathcal{B}$.

Understanding brambles

Definition (Brambles on digraphs)

- Family of strongly connected subgraphs $\mathcal{B} = \{B_1, \ldots, B_\ell\}$
- if $\{B, B'\} \subseteq \mathcal{B}$ then either
  1. $V(B) \cap V(B') \neq \emptyset$ or
  2. edges from B to B' and B' to B.



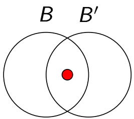


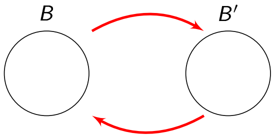
- **hitting set** of $\mathcal{B}$ = set of vertices touching every $B \in \mathcal{B}$.
- **order** of $\mathcal{B}$ = minimum size of hitting set

Understanding brambles

Definition (Brambles on digraphs)

- Family of strongly connected subgraphs $\mathcal{B} = \{B_1, \ldots, B_\ell\}$
- if $\{B, B'\} \subseteq \mathcal{B}$ then either
  1. $V(B) \cap V(B') \neq \emptyset$ or
  2. edges from B to B' and B' to B.

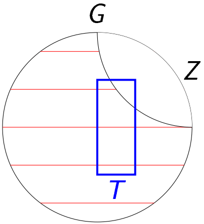




- **hitting set** of $\mathcal{B}$ = set of vertices touching every $B \in \mathcal{B}$.
- **order** of $\mathcal{B}$ = minimum size of hitting set

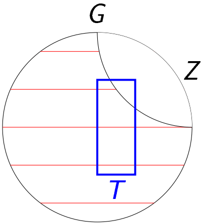
Problem 1

Find compact brambles



- $|T| \leq 2k - 1$.
- (Negative instance)
  No $Z \subseteq V(G)$ satisfies:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong component of $G \setminus Z$.

Solving Problem 1

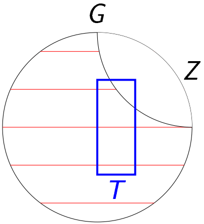


- $|T| \leq 2k - 1$.
- (Negative instance)
  No $Z \subseteq V(G)$ satisfies:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong component of $G \setminus Z$.

Bramble over set T

$\mathcal{B}_T = \{B \subseteq G \mid B$ is induced, strongly connected and $|V(B) \cap T| \geq k\}$.

Solving Problem 1



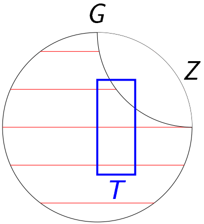
- $|T| \leq 2k - 1$.
- (Negative instance)
  No $Z \subseteq V(G)$ satisfies:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong component of $G \setminus Z$.

Bramble over set T

$\mathcal{B}_T = \{B \subseteq G \mid B$ is induced, strongly connected and $|V(B) \cap T| \geq k\}$.

- $\mathcal{B}$ is a bramble.

Solving Problem 1



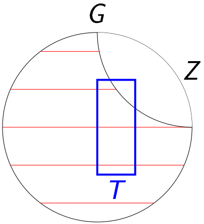
- $|T| \leq 2k - 1$.
- (Negative instance)
  No $Z \subseteq V(G)$ satisfies:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong component of $G \setminus Z$.

Bramble over set T

$\mathcal{B}_T = \{B \subseteq G \mid B$ is induced, strongly connected and $|V(B) \cap T| \geq k\}$.

- $\mathcal{B}$ is a bramble.
- of order k

Solving Problem 1



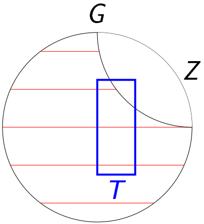
- $|T| \leq 2k-1$.
- (Negative instance)
  No $Z \subseteq V(G)$ satisfies:
  - $|Z| \leq k-1$; and
  - $|C \cap T| \leq k-1$ for strong component of $G \setminus Z$.

Bramble over set T

$\mathcal{B}_T = \{B \subseteq G \mid B$ is induced, strongly connected and $|V(B) \cap T| \geq k\}$.

- $\mathcal{B}$ is a bramble.
- of order k
- Skip havens

Solving Problem 1



- $|T| \leq 2k - 1$.
- (Negative instance)
  No $Z \subseteq V(G)$ satisfies:
  - $|Z| \leq k - 1$; and
  - $|C \cap T| \leq k - 1$ for strong component of $G \setminus Z$.

Bramble over set T

$\mathcal{B}_T = \{B \subseteq G \mid B$ is induced, strongly connected and $|V(B) \cap T| \geq k\}$.

- $\mathcal{B}$ is a bramble.
- of order k
- Skip havens
- Description: T

Objective 2: Key ingredients?

Hitting set path

Poly time: Find a path $P(\mathcal{B}_T)$ which is a hitting set of $\mathcal{B}_T$.

Objective 2: Key ingredients?

Hitting set path

Poly time: Find a path $P(\mathcal{B}_T)$ which is a hitting set of $\mathcal{B}_T$.

Well-linked set

FPT time: If $\mathcal{B}_T$ has order $\frac{k^2}{4} + k$, find a well-linked set in $P(\mathcal{B}_T)$ of size k.

Objective 2: Key ingredients?

Hitting set path

Poly time: Find a path $P(\mathcal{B}_T)$ which is a hitting set of $\mathcal{B}_T$.

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

Well-linked set

FPT time: If $\mathcal{B}_T$ has order $\frac{k^2}{4} + k$, find a well-linked set in $P(\mathcal{B}_T)$ of size k.

Objective 2: Key ingredients?

Hitting set path

Poly time: Find a path $P(\mathcal{B}_T)$ which is a hitting set of $\mathcal{B}_T$.

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$? If not, find disjoint $B \in \mathcal{B}_T$.

Well-linked set

FPT time: If $\mathcal{B}_T$ has order $\frac{k^2}{4} + k$, find a well-linked set in $P(\mathcal{B}_T)$ of size k.

Objective 2: Key ingredients?

Hitting set path

Poly time: Find a path $P(\mathcal{B}_T)$ which is a hitting set of $\mathcal{B}_T$.

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$? If not, find disjoint $B \in \mathcal{B}_T$.

Well-linked set

FPT time: If $\mathcal{B}_T$ has order $\frac{k^2}{4} + k$, find a well-linked set in $P(\mathcal{B}_T)$ of size k.

Key ingredient 2

$\mathcal{B}_T(X, \overline{Y}) = \{B \in \mathcal{B}_T \mid B \text{ intersects } X \text{ and disjoint from } Y\}$

Objective 2: Key ingredients?

Hitting set path

Poly time: Find a path $P(\mathcal{B}_T)$ which is a hitting set of $\mathcal{B}_T$.

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$? If not, find disjoint $B \in \mathcal{B}_T$.

Well-linked set

FPT time: If $\mathcal{B}_T$ has order $\frac{k^2}{4} + k$, find a well-linked set in $P(\mathcal{B}_T)$ of size k.

Key ingredient 2

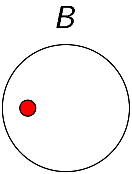
$\mathcal{B}_T(X, \overline{Y}) = \{B \in \mathcal{B}_T \mid B \text{ intersects } X \text{ and disjoint from } Y\}$
FPT time: $\text{order}(\mathcal{B}_T(X, \overline{Y})) \geq k'$?

Using key ingredients

- Find a path P which is hitting set of $\mathcal{B}_T$

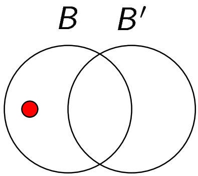
Using key ingredients

- Find a path P which is hitting set of $\mathcal{B}_T$
  - Start with $B \in \mathcal{B}_T$ and $P = v \in B$



Using key ingredients

- Find a path P which is hitting set of $\mathcal{B}_T$
  - Start with $B \in \mathcal{B}_T$ and $P = v \in B$
  - If P does not hit $B' \in \mathcal{B}_T$, improve P (Key ingredient 1)

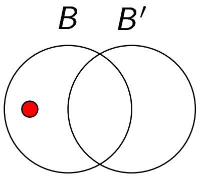


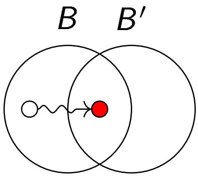
Using key ingredients

- Find a path P which is hitting set of $\mathcal{B}_T$
  - Start with $B \in \mathcal{B}_T$ and $P = v \in B$
  - If P does not hit $B' \in \mathcal{B}_T$, improve P (Key ingredient 1)

Using key ingredients

- Find a path P which is hitting set of $\mathcal{B}_T$
  - Start with $B \in \mathcal{B}_T$ and $P = v \in B$
  - If P does not hit $B' \in \mathcal{B}_T$, improve P (Key ingredient 1)
  - Iterate until hitting set





Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

- EASY: Does $G \setminus Z$ contain a strong component intersecting k vertices of T?

Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

- EASY: Does $G \setminus Z$ contain a strong component intersecting k vertices of T?

Key ingredient 2

FPT time: $\text{order}(\mathcal{B}_T(X, \overline{Y})) \geq k'$?

Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

- EASY: Does $G \setminus Z$ contain a strong component intersecting k vertices of T?

Key ingredient 2

FPT time: order$(\mathcal{B}_T(X, \overline{Y})) \geq k'$?

- Sadly, not so easy...

Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

- EASY: Does $G \setminus Z$ contain a strong component intersecting k vertices of T?

Key ingredient 2

FPT time: order$(\mathcal{B}_T(X, \overline{Y})) \geq k'$?

- Sadly, not so easy...
- $\mathcal{B}_T(\overline{X}, \overline{Y}) = \{B \in \mathcal{B}_T \mid B \text{ disjoint from } X \cup Y\}$

Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

- EASY: Does $G \setminus Z$ contain a strong component intersecting k vertices of T?

Key ingredient 2

FPT time: $\text{order}(\mathcal{B}_T(X, \overline{Y})) \geq k'$?

- Sadly, not so easy...
- $\mathcal{B}_T(\overline{X}, \overline{Y}) = \{B \in \mathcal{B}_T \mid B \text{ disjoint from } X \cup Y\}$
- $\text{order}(\mathcal{B}_T(X, \overline{Y})) + \text{order}(\mathcal{B}_T(\overline{X}, \overline{Y})) \geq \text{order}(\mathcal{B}_T(\overline{Y}))$

Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

- EASY: Does $G \setminus Z$ contain a strong component intersecting k vertices of T?

Key ingredient 2

FPT time: $\text{order}(\mathcal{B}_T(X, \overline{Y})) \geq k'$?

- Sadly, not so easy...
- $\mathcal{B}_T(\overline{X}, \overline{Y}) = \{B \in \mathcal{B}_T \mid B \text{ disjoint from } X \cup Y\}$
- $\text{order}(\mathcal{B}_T(X, \overline{Y})) + \text{order}(\mathcal{B}_T(\overline{X}, \overline{Y})) \geq \text{order}(\mathcal{B}_T(\overline{Y}))$
- $\text{order}(\mathcal{B}_T(\overline{Y}))$ can be solved with techniques similar to $\mathcal{P}$ in $G \setminus Y$

Key ingredient algorithms

Key ingredient 1

Poly time: Is Z a hitting set of $\mathcal{B}_T$?

- EASY: Does $G \setminus Z$ contain a strong component intersecting k vertices of T?

Key ingredient 2

FPT time: order$(\mathcal{B}_T(X, \overline{Y})) \geq k'$?

- Sadly, not so easy...
- $\mathcal{B}_T(\overline{X}, \overline{Y}) = \{B \in \mathcal{B}_T \mid B \text{ disjoint from } X \cup Y\}$
- order$(\mathcal{B}_T(X, \overline{Y}))$ + order$(\mathcal{B}_T(\overline{X}, \overline{Y})) \geq$ order$(\mathcal{B}_T(\overline{Y}))$
- order$(\mathcal{B}_T(\overline{Y}))$ can be solved with techniques similar to $\mathcal{P}$ in $G \setminus Y$
- $\mathcal{B}_T(\overline{X}, \overline{Y}) = \mathcal{B}_T(\overline{X \cup Y})$

Open

- Find directed grid or decomposition in FPT time.

Open

- Find directed grid or decomposition in FPT time.
- Decomposition is supposed to be a tool.

Open

- Find directed grid or decomposition in FPT time.
- Decomposition is supposed to be a tool.
- Most interesting problems are W[1]-hard on graphs of bounded directed tree width.

Open

- Find directed grid or decomposition in FPT time.
- Decomposition is supposed to be a tool.
- Most interesting problems are W[1]-hard on graphs of bounded directed tree width.
- Is there an interesting problem can be shown FPT using this?

Thank you.
Questions?