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Invited talks

Rooted $K_{2,4}$-minors and wye-delta reducibility

Bojan Mohar

Let G be a graph with four distinguished vertices t_1, t_2, t_3, t_4 called terminals. By a rooted $K_{2,4}$ minor in G we refer to a collection of six pairwise disjoint connected subgraphs T_1, T_2, T_3, T_4 and S_1, S_2 of G such that $t_i \in V(T_i)$ for $1 \leq i \leq 4$ and each T_i is adjacent to S_1 and S_2.

A result about existence of rooted $K_{2,4}$ minors in planar graphs and the corresponding structure theorem will be presented. Although the proof is both long and complicated, the structure turns out to be quite accessible.

A graph with four terminals is wye-delta reducible if we can obtain a graph on four vertices by a sequence of wye-delta operations and series-parallel reductions, none of which is allowed to remove any of the terminals. In the second part of the talk we shall apply the result about rooted $K_{2,4}$ minors to derive a characterization of 4-terminal wye-delta reducibility in planar graphs.

Joint work with Lino Demasi.

The importance of being simple

János Pach

Given n segments in the plane, it follows by Ramsey's theorem that one can always find roughly $\log n$ among them that are pairwise disjoint or pairwise intersecting. The truth is much better: it can be shown that one can select at least n^{ϵ} segments with the above property. Given n points and n lines in the plane, their incidence graph contains no $K_{2,2}$. This implies, using the Kővári-Sós-Turán-Erdős theorem, that the number of incidences cannot exceed $n^{3/2}$. The Szemerédi-Trotter theorem states, however, that the true order of magnitude of the maximum number of incidences between n points and n lines is smaller: $n^{4/3}$. Are our combinatorial tools too weak to tackle these and many other geometric intersection problems? In this talk, we illustrate how to improve three cornerstones of extremal combinatorics, (1) Ramsey's theorem, (2) Turán-type theorems, and (3) Szemerédi's regularity lemma, for "algebraically defined" graphs and hypergraphs.

Paul Seymour

Improved Approximations for Graph-TSP in Regular Graphs
R. Ravi

A tour in a graph is a connected walk that visits every vertex at least once, and returns to the starting vertex. We give improved approximation results for a tour with the minimum number of edges in regular graphs.

For cubic bipartite graphs, we provide a polynomial-time (9/7)-approximation algorithm for minimum tours. For connected d-regular graph with n vertices, we provide a method that constructs a tour of length at most $(1 + O(1/\sqrt{d}))n$, improving the previous result of Vishnoi (2012) that demonstrated a tour of length at most $(1 + O(1/\sqrt{\log d}))n$.

The former result uses the cubic bipartite graph structure to find a cycle cover with large average length. The latter finds a spanning tree with few odd-degree vertices and augments it to a tour. Finding such spanning trees to augment is related to the linear arboricity conjecture of Akiyama, Exoo and Harary (1981), or alternatively, to a conjecture of Magnant and Martin (2009) regarding the path cover number of regular graphs.

Joint work with Uriel Feige, Jeremy Karp, and Mohit Singh.

The Structure and χ-Boundedness of Typical Graphs in a Hereditary Family
Bruce Reed

Some of the most important results and open questions in graph colouring, including Berge's Celebrated Strong Perfect Graph Conjecture, have the following form:

Given a hereditary family $\mathcal{F}$ of graphs (i.e. a family closed under the taking of induced subgraphs) what is the relationship of the chromatic number of a graph G in $\mathcal{F}$ to the size of its largest clique.

We focus on a related question: Given a hereditary family $\mathcal{F}$ of graphs, what is the relationship of the chromatic number of a typical graph G in $\mathcal{F}$ to the size of its largest clique.

The results presented were obtained in collaboration with Kang, McDiarmid, Scott, and Yuditsky.

Every graph is easy or hard: dichotomy theorems for graph problems
Dániel Marx

Given a family of algorithmic problems, a dichotomy theorem characterizes each member of the family either as "easy" or as "hard." A classical example is the result of Hell and Nešetřil classifying the complexity of H-Coloring for every fixed H: it is polynomial-time solvable if H is bipartite and NP-hard for *every* nonbipartite graph. Some dichotomy theorems characterize the complexity of a family of problems in a more general setting, where a problem in the family is defined not just by fixing a single graph H, but by fixing a (potentially infinite) *class* of graphs. For example, a result of Yannakakis characterizes the complexity of node deletion problems for *any* hereditary class of graphs, while a result of Grohe characterizes the complexity of graph homomorphisms when the left-hand side graph is restricted to be a member of a fixed class of graphs. In the talk, we survey classical and recent dichotomy theorems arising in the context of graph problems.

Flows in matroids
Bertrand Guenin

In an undirected graph with integer capacities the Max-Flow Min-Cut theorem states that the maximum integer flow between a fixed pair of vertices is equal to the minimum capacity of any cut separating these vertices. Flows in graphs generalize naturally to binary matroids, however, the analogous minimax relation does not hold in general. Over thirty years ago Seymour proposed the Cycling and Flowing conjectures. The former (resp. latter) would give sufficient conditions for the existence of an integer (resp. fractional) flow in a binary matroid. I will present recent progress on both of these conjectures. These results generalize several classical minimax theorems on graphs. This is joint work with one of my student Ahmad Abdi.

Easily testable graph properties
Noga Alon

A graph on n vertices is ϵ-far from a property $\mathcal{P}$ if one has to add or delete from it at least ϵn^2 edges to get a graph satisfying $\mathcal{P}$. A graph property is easily testable if it is possible to distinguish between graphs satisfying $\mathcal{P}$ and ones that are ϵ-far from $\mathcal{P}$ by inspecting the induced subgraph on a random subset of poly$(1/\epsilon)$ vertices. I will consider the problem of characterizing the easily testable graph properties, which is wide open, describe its background and history, and report on some (modest) recent progress in its study in joint work with Jacob Fox.

Computational Topology of Cuts and Flows
Jeff Erickson

The maximum flow and minimum cut problems have been targets of intense algorithmic research for more than half a century.

Flows and cuts in planar graphs have been studied from the very beginning of this history; however, until recently, relatively little was known about flows and cuts in even slightly more general classes of graphs. This talk will survey recent algorithms for computing maximum flows and minimum cuts in planar and surface-embedded graphs in near-linear time. The key insight for surface algorithms is to view flows and cuts through the lens of homology, a standard tool in algebraic topology introduced more than 100 years ago by Henri Poincaré. Surprisingly, topology also offers a clean framework to describe efficient flow algorithms even for planar graphs.

This talk includes joint work with Erin Wolf Chambers, Kyle Fox, and Amir Nayyeri.

Contributed talks

Induced cycles in triangle graphs

Aparna Lakshmanan S. (joint work with Csilla Bujtás and Zsolt Tuza)

The triangle graph of a graph G, denoted by $\mathcal{T}(G)$, is the graph whose vertices represent the triangles of G, and two vertices of $\mathcal{T}(G)$ are adjacent if and only if the corresponding triangles of G share an edge. In this paper, we characterize graphs G whose triangle graph is a cycle and then extend the result to the characterization of C_n-free triangle graphs. Using this, we characterize graphs G whose $\mathcal{T}(G)$ is a tree, a chordal graph and a perfect graph. In the last section, we prove a conjecture of the third author regarding packing and covering the triangles of a graph, for the class of graphs whose triangle graph is perfect.

On the Overlap Number of Chordal and Interval Graphs

Claudson F. Bornstein (joint work with Rafael O. Lopes, Márcia R. Cerioli, and Jayme L. Szwarcfiter)

The overlap number of a graph is the size of the smallest number of elements needed obtain sets corresponding to each vertex so that two vertices are adjacent if the corresponding sets overlap. In this paper we present an algorithm that produces an overlap representation for a chordal graph G with at most $n+l(G)-2$ elements, where $l(G)$ is the leafage of the graph. For interval graphs this bound becomes n which is the best possible. However, we do not know how far the algorithm can be from the optimum. We also obtain a lower bound on the overlap number in terms of the asteroidal number of the graph.

A Characterization of Mixed Unit Interval Graphs
Felix Joos

We give a complete characterization of mixed unit interval graphs, the intersection graphs of closed, open, and half-open unit intervals of the real line. This is a proper superclass of the well known unit interval graphs. Our result solves a problem posed by Dourado, Le, Protti, Rautenbach and Szwarcfiter (Discrete Math. **312** (2012), 3357-3363). Our characterization also leads to a polynomial-time recognition algorithm for mixed unit interval graphs.

Perfectly relating the Domination, Total Domination, and Paired Domination Numbers of a Graph
José D. Alvarado (joint work with Simone Dantas and Dieter Rautenbach)

The domination number $\gamma(G)$, the total domination number $\gamma_t(G)$, the paired domination number $\gamma_p(G)$, the domatic number $d(G)$, and the total domatic number $d_t(G)$ of a graph G without isolated vertices are related by trivial inequalities $\gamma(G) \leq \gamma_t(G) \leq \gamma_p(G) \leq 2\gamma(G)$ and $d_t(G) \leq d(G)$. Very little is known about the graphs that satisfy one of these inequalities with equality. We study classes of graphs defined by requiring equality in one of the above inequalities for all induced subgraphs that have no isolated vertices and whose domination number is not too small. Our results are characterizations of several such classes in terms of their minimal forbidden induced subgraphs. Furthermore, we prove some hardness results, which suggest that the extremal graphs for some of the above inequalities do not have a simple structure.

Quasiperfect Dominating Codes in Graphs
Ignacio M. Pelayo (joint work with José Cáceres, Carmen Hernando, Mercè Mora, and M.L. Puertas)

Given a graph G, a set $D \subseteq V(G)$ is a *dominating set* of G if every vertex not in D is adjacent to at least one vertex of D. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. A dominating set of cardinality $\gamma(G)$ is called a γ*-code.*

If moreover, every vertex not in D is adjacent to exactly one vertex of D, then D is called a *perfect dominating set* of G. The *perfect domination number* $\gamma_{11}(G)$ is the minimum cardinality of a perfect dominating set of G. A dominating set of cardinality $\gamma_{11}(G)$ is called a γ_{11}*-code.*

For every integer $k \geq 1$, a dominating set D is called a k*-quasiperfect dominating set* if every vertex not in D is adjacent to at most k vertices of D. The k*-quasiperfect domination number* $\gamma_{1k}(G)$ is the minimum cardinality of a k-quasiperfect dominating set of G. A dominating set of cardinality $\gamma_{1k}(G)$ is called a γ_{1k}*-code.*

Certainly, 1-quasiperfect dominating sets and Δ-quasiperfect dominating sets are precisely the perfect dominating sets and dominating sets, respectively, where Δ stands for the maximum degree of the graph. It also clear that, if G is a graph of order n and maximum degree Δ, then:

$$n \geq \gamma_{11}(G) \geq \gamma_{12}(G) \geq \ldots \geq \gamma_{1\Delta}(G) = \gamma(G).$$

Our work consists basically in studying this decreasing chain of domination parameters in a number of different ways. In this talk, we present both the state of art and our main contributions when restricting ourselves to the following graph families: graphs with maximum degree $\Delta \geq n-3$, graphs with maximum degree $\Delta \leq 3$, cographs, claw-free graphs, trees, Cartesian product graphs and strong product graphs.

Domination, Eternal Domination, and Clique Covering
Kieka Mynhardt (joint work with William F. Klostermeyer)

Eternal and m-eternal domination are concerned with using mobile guards to protect a graph against infinite sequences of attacks at vertices. Eternal domination allows one guard to move per attack, whereas more than one guard may move per attack in the m-eternal domination model. We explore inequality chains consisting of the domination, eternal domination, m-eternal domination, independence, and clique covering numbers of a graph.

We characterize triangle-free graphs with domination and eternal domination numbers equal to two, trees with equal m-eternal domination and clique covering numbers, and two classes of graphs with equal domination, eternal domination and clique covering numbers.

A Lower Bound on the Crossing Number of Uniform Hypergraphs
Saswata Shannigrahi (joint work with Anurag Anshu)

In this paper, we consider the embedding of a complete d-uniform geometric hypergraph with n vertices in general position in $\mathbb{R}^d$, where each hyperedge is represented as a $(d-1)$-simplex, and a pair of hyperedges is defined to cross if they are vertex-disjoint and contains a common point in the relative interior of the simplices corresponding to them. As a corollary of the Van Kampen-Flores Theorem, it can be seen that such a hypergraph contains $\Omega(\frac{2^d}{\sqrt{d}})$ $\binom{n}{2d}$ crossing pairs of hyperedges. Using Gale Transform and Ham Sandwich Theorem, we improve this lower bound to $\Omega(\frac{2^d \log d}{\sqrt{d}})$ $\binom{n}{2d}$.

The colourful simplicial depth conjecture
Pauline Sarrabezolles

Given $d+1$ sets of points, or colours, $\mathbf{S}_1, \ldots, \mathbf{S}_{d+1}$ in $\mathbb{R}^d$, a *colourful simplex* is a set $T \subseteq \bigcup_{i=1}^{d+1} \mathbf{S}_i$ such that $|T \cap \mathbf{S}_i| \leq 1$, for all $i \in \{1, \ldots, d+1\}$. The colourful Carathéodory theorem states that, if $\mathbf{0}$ is in the convex hull of each $\mathbf{S}_i$, then there exists a colourful simplex T containing $\mathbf{0}$ in its convex hull. Deza, Huang, Stephen, and Terlaky conjectured in 2006 that, when $|\mathbf{S}_i| = d+1$ for all $i \in \{1, \ldots, d+1\}$, there are always at least d^2+1 colourful simplices containing $\mathbf{0}$ in their convex hulls. We prove this conjecture via a combinatorial approach.

On the separation of Tverberg partitions of large sets of points

Natalia García-Colín

A set $X \subset \mathbb{R}^d$ of points in general position is (k,r)-*separable* if for all partitions of X into r disjoint sets, $X = \bigcup_1^r A_r$ there is a set $K \subset X$, such that $|K| = k$ and $\bigcap_1^r conv(A_r \setminus K) = \emptyset$. Let $\kappa(X,d,r)$ be the minimum number such that X is (κ, r)-separable and define the *minimum separability number* for some d,n,r as $\kappa(d,n,r) = \min_{\{X \mid X \subset \mathbb{R}^d,\ |X|=n\}} \kappa(X,d,r)$. We prove that $\lim_{n\to\infty} \frac{\kappa(n,d,r)}{n} = \frac{1}{r}$.

Furthermore if we consider the (n,d,r)-*separability number*, defined as $K(d,n,r) = \max_{\{X \mid X \subset \mathbb{R}^d,\ |X|=n\}} \kappa(X,d,r)$ we exhibit a construction that shows that even for all relatively small n as compared to r, $K(d,n,r)$ is very near $\frac{n}{r}$.

Covering Intersecting Bi-set Families Under Matroid Constraints

Tamás Király (joint work with Kristóf Bérczi and Yusuke Kobayashi)

Edmonds' fundamental theorem on arborescences characterizes the existence of k pairwise edge-disjoint arborescences with the same root in a directed graph. Lovász gave an elegant alternative proof which became the base of many extensions of Edmonds' result. Recent developments include results of Durand de Gevigney et al. on packing arborescences under matroid constraints.

Frank observed that Edmonds' weak theorem can be reformulated in terms of covering an intersecting set family k times. His approach was further generalized by Szegő, and also extended to biset families by Bérczi and Frank. Hence the natural question arises: is there a common generalization of these research directions?

We use a modification of Lovász' method to prove a theorem on covering intersecting bi-set families under matroid constraints. Our result can be considered as a common generalization of previous results on packing arborescences and on covering intersecting bi-set families.

Zigzags on Greedoids
Yulia Kempner (joint work with Vadim E. Levit)

Pivoting, i.e. exchanging exactly one element in a basis, is a fundamental step in the simplex algorithm for linear programming. A combinatorial analog of this operation is defined on the bases of a greedoid. We extend this definition to all feasible sets of the same cardinality and obtain new characterizations of antimatroids and matroids.

Matroid union, Graphic? Binary? Neither?
Csongor Gy. Csehi (joint work with András Recski)

There is a conjecture that if the union (also called sum) of graphic matroids is not graphic then it is nonbinary. Some special cases have been proved only, for example if several copies of the same graphic matroid are given. If there are two matroids and the first one can either be represented by a graph with two points, or is the direct sum of a circuit and some loops, then a necessary and sufficient condition is given for the other matroid to ensure the graphicity of the union. These conditions can be checked in polynomial time. The proofs imply that the above conjecture holds for these cases.

Minimum Size Tree-Decompositions
Bi Li (joint work with Fatima Zahra Moataz and Nicolas Nisse)

Tree-Decompositions are the corner-stone of many dynamic programming algorithms for solving graph problems. Since the complexity of such algorithms generally depends exponentially on the *width* (size of the *bags*) of the decomposition, much work has been devoted to compute tree-decompositions with small width. However, practical algorithms computing tree-decompositions only exist for graphs with *treewidth* less than 4. In such graphs, the time-complexity of dynamic programming algorithms based on tree-decompositions is dominated by the *size* (number of bags) of the tree-decompositions. It is then interesting to try to minimize the size of the tree-decompositions.

In this extended abstract, we consider the problem of computing a tree-decomposition of a graph with width at most k and minimum size. More precisely, we focus on the following problem: given a fixed $k \geq 1$, what is the complexity of computing a tree-decomposition of width at most k with minimum size in the class of graphs with treewidth at most k? We prove that the problem is NP-complete for any fixed $k \geq 4$ and polynomial for $k \leq 2$. Ongoing work also suggests it is polynomial for $k = 3$.

On the Minimum Bisection of Graphs with Low Tree Width
Tina Janne Schmidt (joint work with Cristina G. Fernandes and Anusch Taraz)

Minimum Bisection denotes the NP-hard problem to partition the vertex set of a graph into two sets of equal sizes while minimizing the number of edges between these two sets. We consider this problem in bounded degree graphs with a given tree decomposition and prove an upper bound for their minimum bisection width in terms of the width and the structure of the provided tree decomposition. If the tree decomposition satisfies certain properties, we can find a corresponding bisection in $O(nt)$ time, where n denotes the number of the vertices of the graph and $t-1$ is the width of the provided tree decomposition.

Subdivisions in 4-connected graphs of large tree-width
Irene Muzi (joint work with Paul Wollan)

We prove that 4-connected graphs of sufficiently large treewidth contain a subdivision of either a large grid or a graph obtained by adding an apex vertex to a 3-regular graph of large treewidth. Using analogous techniques we prove that nonplanar 4-connected graphs of sufficiently large treewidth contain K_5 as a subdivision.

The rank of a graph convexity: complexity aspects

Vinícius F. dos Santos (joint work with Igor da Fonseca Ramos and Jayme L. Szwarcfiter)

In this work we introduce the study of the complexity of computing the rank of a graph convexity. We consider the P_3-convexity and the monophonic convexity, in which the convex sets are closed for paths of order 3 and induced paths, respectively.

The convex hull $H(S)$ of a set S is the smallest convex set containing S as a subset. A set S is a convexly independent set if $v \notin H(S \setminus \{v\})$ for all v in S. The rank of a graph is the size of the largest convexly independent set. In this work we consider the problem of determining the rank of a graph.

For the P_3-convexity, we show that the problem is NP-complete even for split or bipartite graphs. We also show how to determine $rk(G)$ in polynomial time for the well behaved classes of graphs of trees and threshold graphs and give an upper bound for $rk(G)$. Finally, we show some implications of our results to the open packing number of a graph, studied in the context of domination.

For the monophonic convexity, we show the NP-completeness even for graphs without a separating clique.

Chordal-$(2, 1)$ graph sandwich problem with boundary conditions

Fernanda Couto (joint work with Luerbio Faria, Sylvain Gravier, and Sulamita Klein)

The original graph sandwich problem for a property Π, as defined by Golumbic, Kaplan, and Shamir, can be stated as follows: given two graphs $G^1 = (V, E^1)$ and $G^2 = (V, E^2)$, is there a graph $G = (V, E)$ such that $E^1 \subseteq E \subseteq E^2$ and G satisfies Π? The graph

G, if it exists, is called a *sandwich graph.* Graph sandwich problems for property Π generalize recognition problems for the same property, by setting $G^1 = G^2$. Our proposal is to introduce a generalization of the original graph sandwich problem, by specifying properties Π_1, Π_2 for graphs G^1, G^2, respectively. In this new approach, each type of graph sandwich problem can be represented by a triple (Π^1, Π, Π^2)-SP, whose meaning is precisely "seek for a sandwich graph G satisfying Π, when it is known that G^i satisfies Π^i, $i = 1, 2$". Such a generalization is called *graph sandwich problem with boundary conditions.* When G^i is not required to satisfy any property, we denote Π^i by $*$. One of the motivations for introducing boundary conditions is to develop a more refined complexity analysis of a NP-complete $(*, \Pi, *)$ problem, since its complexity status can change to polynomially solvable by suitably selecting Π^1, Π^2. For instance, it is known that $(*, chordal - (2, 1), *)$-SP is NP-complete. In this work we prove that this problem can be solved in polynomial time when choosing G^2 a graph in a family in which the number of maximal cliques is polynomial.

Nonempty Intersection of Longest Paths in Partial 2-Trees
Carl Georg Heise (joint work with Julia Ehrenmüller and Cristina G. Fernandes)

In 1966 Gallai asked whether all longest paths in a connected graph have nonempty intersection. This is not true in general and various counterexamples have been found. However, the answer to Gallai's question is positive for several well-known classes of graphs, e. g. outerplanar graphs, split graphs, and 2-trees. We present a proof that all connected subgraphs of 2-trees (also called series-parallel graphs) have a vertex that is common to all of its longest paths. Since outerplanar graphs and in particular 2-trees are subgraphs of 2-trees, our result captures these two classes in one proof and strengthens them to a larger class of graphs.

Edge criticality in secure graph domination
Jan van Vuuren (joint work with Alewyn Burger and Anton de Villiers)

A subset X of the vertex set of a graph G is a secure dominating set of G if X is a dominating set of G and if, for each vertex u not in X, there is a neighbouring vertex v of u in X such that the swap set $(X\backslash\{u\})\cup\{v\}$ is again a dominating set of G. The secure domination number of G is the cardinality of a smallest secure dominating set of G. A graph G is q-critical if the smallest arbitrary subset of edges whose removal from G necessarily increases the secure domination number, has cardinality q. In this paper we characterise q-critical graphs for all admissible values of q and determine the exact values of q for which members of various infinite classes of graphs are q-critical.

Equality in a Bound that Relates the Size and the Restrained Domination Number of a Graph
Johannes H. Hattingh (joint work with Ernst J. Joubert)

Let $G = (V, E)$ be a graph. A set $S \subseteq V$ is a restrained dominating set if every vertex in $V - S$ is adjacent to a vertex in S and to a vertex in $V - S$. The restrained domination number of G, denoted $\gamma_r(G)$, is the smallest cardinality of a restrained dominating set of G. For $n \geq 1$ and $k \in \{1, \ldots, n-2, n\}$, let

$$q(n,k) = \begin{cases} \binom{n}{2} & \text{if } 1 = k \leq n \text{ and } n \geq 4, \\ \frac{n(n-2)}{2} & \text{if } 2 = k \leq n \text{ and } n \geq 6 \text{ is even}, \\ \frac{n(n-2)-1}{2} & \text{if } 2 = k \leq n \text{ and } n \geq 5 \text{ is odd}, \\ \binom{n-k}{2} + n + \frac{n-k}{2} - 3 & \text{if } 3 \leq k \leq n-3 \text{ and } n-k \text{ is even}, \\ \binom{n-k}{2} + n + \frac{n-k+1}{2} - 3 & \text{if } 3 \leq k \leq n-3 \text{ and } n-k \text{ is odd}, \\ n & \text{if } 1 \leq k = n-2, \\ n-1 & \text{if } 1 \leq k = n. \end{cases}$$

It was shown that if G is a graph of order n with $\gamma_r(G) = k \in \{3, \ldots, n-2\}$, then $m(G) \leq q(n,k)$. In this paper, we extend this result by showing that if G is a graph of order $n \geq 1$ and $k \in \{1, \ldots, n\}$, then $m(G) \leq q(n,k)$. We also characterize graphs G of order n with $\gamma_r(G) = k \in \{1, \ldots, n\}$ for which $m(G) = q(n,k)$.

Domination in designs

Felix Goldberg (joint work with Rogers Mathew and Deepak Rajendraprasad)

We commence the study of domination in the incidence graphs of combinatorial designs. Let D be a combinatorial design and denote by $\gamma(D)$ the domination number of the incidence (Levy) graph of D. We obtain a number of results about the domination numbers of various kinds of designs.

For instance, a finite projective plane of order n, which is a symmetric $(n^2+n+1, n+1, 1)$-design, has $\gamma = 2n$. We study at depth the domination numbers of Steiner systems and in particular of Steiner triple systems. We show that a $STS(v)$ has $\gamma \geq \frac{2}{3}v - 1$ and also obtain a number of upper bounds. The tantalizing conjecture that all Steiner triple systems on v vertices have the same domination number is proposed. So far it has been verified up to $v \leq 15$.

The structure of minimal dominating sets is also investigated, both for its own sake and as a tool in deriving lower bounds on γ. Finally, a number of open questions are proposed.

Symplectic graphs over finite commutative rings

Yotsanan Meemark (joint work with Thammanoon Puirod)

This work gives some further developments of the symplectic graph $\mathcal{G}_{\mathrm{Sp}_R(V)}$, where V is a symplectic space over a finite commutative ring R. We can classify if our graph is a strongly regular graph or a d-Deza graph. We show that it is arc transitive, and determine the chromatic numbers.

On Hamiltonian cycle systems with non trivial automorphism group

Gloria Rinaldi

A Hamiltonian cycle system (HCS for short) of a graph Γ is a collection of Hamiltonian cycles whose edges partition the edge set of Γ. When speaking of an HCS of order v (briefly an HCS(v)) we mean $\Gamma = K_v$ (the complete graph on v vertices) or $\Gamma = K_v - I$ (the complete graph on v vertices with one $1-$factor I removed) according to whether v is odd or even, respectively. An automorphism of an HCS is a bijection on its vertex-set leaving it invariant. It seems that the most successful way to find "many" pairwise non isomorphic HCSs of the same order is to assume that they are $1-$rotational under some group G (namely, admitting G as an automorphism group acting sharply transitively on all but one vertex) when v is odd, or to assume that they are $2-$pyramidal under some group G (namely, admitting G as an automorphism group fixing 2 vertices and acting sharply transitively on the others) when v is even. I focus the attention on some recent results concerning $1-$rotational and $2-$pyramidal HCSs.

Construction of chiral 4-polytopes with alternating or symmetric automorphism group

Eugenia O'Reilly-Regueiro (joint work with Marston Conder, Isabel Hubard, and Daniel Pellicer)

We describe a construction for finite abstract chiral 4-polytopes with Schläfli type $\{3, 3, k\}$ (with tetrahedral facets), and with an alternating or symmetric group as automorphism group. We use it to prove that for all but finitely many n, both A_n and S_n are the automorphism groups of such a polytope. We also show that the vertex-figures of the polytopes obtained from our construction are chiral.

Matroid Theory for Metabolic Network Analysis

Arne C. Reimers (joint work with Leen Stougie)

Recently, we introduced *flux modules* into the area of metabolic network analysis to characterize substructures in metabolic networks formed by chemical reactions inside biological cells. Outside of metabolic network analysis, a generalization of flux modules called *k-modules* can be used to address the vertex enumeration problem of polytopes in computational discrete geometry.

In this talk, we will show how k-modules can be characterized using matroid connectivity. This way, we can link branch-decompositions to structural properties of metabolic networks and use them to develop algorithms in computational biology and discrete geometry. In particular, we will show how vertices of polytopes can be enumerated in total polynomial time if the branch-width is bounded by a constant.

We think that metabolic networks offer an interesting application area of matroid theory with many open problems that can possibly be solved elegantly with matroid theory.

Detecting minors in matroids through triangles

Boris Albar (joint work with Daniel Gonçalves and Jorge L. Ramírez Alfonsín)

In this note we investigate some matroid minor structure results. In particular, we present sufficient conditions, in terms of *triangles*, for a matroid to have either $U_{2,4}$ or F_7 or $M(K_5)$ as a minor.

Cofinitary transversal matroids

Hadi Afzali (joint work with Hiu Fai Law and Malte Müller)

Transversal matroids are usually given via their presentations (which are bipartite graphs). As graph properties are easy to visualize, it is often convenient to explore the matroid via the bipartite graphs presenting it.

In 2013, Carmesin characterized presentations of finitary strict gammoids (*finitary* matroids are the ones with no infinite circuit) via forbidden topological minors. We combine his result with duality results in Afzali et al. (2014) to characterize presentations of cofinitary transversal matroids (*cofinitary* matroids are duals of finitary ones) among all transversal matroids' presentations.

Well-quasi-ordering graphs by the topological minor relation

Chun-Hung Liu (joint work with Robin Thomas)

Robertson and Seymour proved that graphs are well-quasi-ordered by the minor relation and the weak immersion relation. That is, given infinitely many graphs, one graph contains another as a minor or a weak immersion, respectively. However, the topological minor relation does not well-quasi-ordered graphs in general. Robertson conjectured in 1980's that for every positive integer k, graphs that do not contain a topological minor isomorphic to the path of length k with every edge duplicated are well-quasi-ordered by the topological minor relation. We prove this conjecture in this paper. This generalizes the known results that topological minor relation well-quasi-orders the graphs containing bounded number of disjoint cycles and the subcubic graphs. Furthermore, our result leads to the existence of polynomial time algorithms for testing topological minor-closed properties for certain classes of graphs.

Structure of W_4-immersion free graphs

Rémy Belmonte (joint work with Archontia C. Giannopoulou, Daniel Lokshtanov, and Dimitrios M. Thilikos)

We study the structure of graphs that do not contain the wheel on 5 vertices W_4 as an immersion, and show that these graphs can be constructed via 1, 2, and 3-edge-sums from subcubic graphs and graphs of bounded treewidth.

Tree-depth and Vertex-minors
Jan Obdržálek (joint work with Petr Hliněný, O-joung Kwon, and Sebastian Ordyniak)

In a recent paper, Kwon and Oum claim that every graph of bounded rank-width is a pivot-minor of a graph of bounded tree-width (while the converse has been known true already before). We study the analogous questions for "depth" parameters of graphs, namely for the tree-depth and related new shrub-depth. We show that shrub-depth is monotone under taking vertex-minors, and that every graph class of bounded shrub-depth can be obtained via vertex-minors of graphs of bounded tree-depth. We also consider the same questions for bipartite graphs and pivot-minors.

Parity Tournaments of Planar Point Sets
Guillaume Guégan

To a set P of points in general position in the plane, with $|P|$ odd, we associate a tournament, which we call the parity tournament of P: the vertices are the points of P, and p dominates q in the tournament if the directed line $\overrightarrow{pq}$ in the plane has an even number of points on its right side. Tournaments obtained this way are regular, but not every regular tournament is a parity tournament. A regular tournament that is a parity tournament is said to be realizable. We prove realizability for some small classes of tournaments. We then generalize parity tournaments to the setting of rank 3 uniform oriented matroid, and draw an interesting parallel between 3-circuits of the former, and mutations of the latter. We conjecture that every regular tournament is the parity tournament of an uniform oriented matroid.

Complementary cycles in regular bipartite tournaments
Yandong Bai (joint work with Hao Li and Weihua He)

We show that every k-regular bipartite tournament B with $k \geq 3$

has two complementary cycles of lengths 6 and $|V(B)|-6$, unless B is isomorphic to a special digraph.

On the clique structure of edge intersection graphs of subtrees of a tree
Elad Cohen (joint work with Eli Berger and Irith Ben-Arroyo Hartman)

The family of *Edge intersection graphs of Paths in a Tree (EPT)* was introduced by Golumbic and Jamison. They showed that the recognition and the coloring problems for this class are NP-complete. However, they presented a polynomial time solution for the maximum clique problem for EPT graphs, by characterizing their clique structure. We study the clique structure of *Edge intersection graphs of Subtrees of a Tree (EST)*, a natural generalization of EPT graphs. In particular, we study EST graphs where the maximum degree of the subtrees is bounded, namely EST_k graphs. We introduce a geometrical model, namely *semi-projective planes*, which generalizes the known model of projective planes. We show that the problem of characterizing cliques in EST_k graphs is equivalent to the problem of finding the maximum number of points in a semi-projective plane. We give exponential lower and upper bounds for the maximum number of points in a semi-projective plane of order k, for $k \geq 1$, and tight bounds for the cases $k = 1$ and $k = 2$.

Graph products of the trivariate total domination polynomial
Markus Dod

A vertex subset $W \subseteq V$ of the graph $G = (V, E)$ is a total dominating set if every vertex of the graph is adjacent to at least one vertex in W. The total domination polynomial is the ordinary generating function for the number of total dominating sets in the graph. We investigate some graph products for a generalization of the total domination polynomial, called the trivariate total domination polynomial. These results have a wide applicability to other domination related graph polynomials, e.g. the domination polynomial, the independent domination polynomial or the independence polynomial.

On the Number of Minimal Dominating Sets on Cobipartite and Interval Graphs
Romain Letourneur (joint work with Jean-François Couturier and Mathieu Liedloff)

A dominating set in a graph is a subset of vertices such that each vertex is either in the dominating set or adjacent to some vertex in the dominating set. It is known that graphs have at most $O(1.7159^n)$ minimal dominating sets. Here we establish upper bounds on this maximum number of minimal dominating sets for cobipartite and interval graphs. For each of these graph classes, we provide an algorithm to enumerate them. For interval graphs, we show that the number of minimal dominating sets is at most $3^{n/3} \approx 1.4423^n$, which is the best possible bound. For cobipartite graphs, we lower the $O(1.5875^n)$ upper bound from Couturier to $O(1.4511^n)$.

Dominating induced matching in subcubic $S_{2,2,2}$-free graphs
Alain Hertz (joint work with Vadim Lozin and Bernard Ries)

We study the problem of determining whether or not a graph G has an induced matching that dominates every edge of the graph, which is also known as efficient edge domination. This problem is known to be NP-complete in general as well as in some restricted domains, such as bipartite graphs or regular graphs. In this paper, we prove that this problem is solvable in polynomial time if G is subcubic and $S_{2,2,2}$-free, where $S_{2,2,2}$ is a tree with exactly three vertices of degree one, all of them being at distance two from the only vertex of degree three.

Induced Matchings in Subcubic Graphs
Thomas Sasse (joint work with Felix Joos and Dieter Rautenbach)

We prove that a cubic graph with m edges has an induced matching with at least $m/9$ edges. Our result generalizes a result for planar graphs due to Kang, Mnich, and Müller and solves a conjecture of Henning and Rautenbach.

Bounds and algorithms for limited packings in graphs
Andrei Gagarin (joint work with Vadim Zverovich)

We consider (closed neighbourhood) packings and their generalization in graphs called limited packings. A vertex set X in a graph G is a *k-limited packing* if for any vertex $v \in V(G)$, $|N[v] \cap X| \leq k$, where $N[v]$ is the closed neighbourhood of v. The *k-limited packing number* $L_k(G)$ is the largest size of a k-limited packing in G. Limited packing problems can be considered as secure facility location problems in networks. We develop probabilistic and greedy approaches to limited packings in graphs, providing lower bounds for the k-limited packing number, and randomized and greedy algorithms to find k-limited packings satisfying the bounds. Some upper bounds for $L_k(G)$ are given as well. The problem of finding a maximum size k-limited packing is known to be NP-complete even in split or bipartite graphs.

Packing grids into complete graphs
Silvia Messuti (joint work with Vojtěch Rödl and Mathias Schacht)

Motivated by a conjecture of Gyárfás, recently Böttcher, Hladký, Piguet, and Taraz showed that every collection $T_1, \ldots, T_n$ of trees on n vertices with $\sum_{i=1}^{n} e(T_i) \leq \binom{n}{2}$ and with bounded maximum degree, can be packed (edge disjointly) into the complete graph on $(1 + o(1))n$ vertices. We found a different proof of this result, which extends to other graphs than trees. Here we present the proof for the special and somewhat simpler case, when the graphs of the packing are grid graphs.

Labeled embedding of $(n, n-2)$ graphs in their complements
Éric Duchêne (joint work with Hamamache Kheddouci, Mohammed Amin Tahraoui and Marius Woźniak)

Graph packing generally deals with unlabeled graphs. In 2011, Duchêne et al. introduced a new variant of the graph packing problem, called *labeled packing of a graph*. In this paper, we present some results on the labeled packing number of two copies of $(n, n-2)$ graphs into K_n.

Generalized multiplicative Sidon-sequences
Péter Pál Pach

As a generalization of multiplicative Sidon-sequences we investigate the following question: What is the maximal number of elements which can be chosen from the set $\{1, 2, \ldots, n\}$ in such a way that the equation $a_1 a_2 \ldots a_k = b_1 b_2 \ldots b_k$ does not have a solution of distinct elements? Let us denote this maximal number by $G_k(n)$. Erdős studied the case $k = 2$: In 1938 he proved that $\pi(n) + c_1 n^{3/4}/(\log n)^{3/2} \leq G_2(n) \leq \pi(n) + c_2 n^{3/4}$ and 31 years later he improved the upper bound to $\pi(n) + c_2 n^{3/4}/(\log n)^{3/2}$. Hence, in the lower- and upper bounds for $G_2(n)$ not only the main terms are the same, but the error terms only differ by a constant factor. We study $G_k(n)$ for $k > 2$, give asymptotically precise bounds for every k, and prove some estimates on the error terms.

To estimate $G_k(n)$ extremal graph theoretic results are used, namely results about the maximal number of edges of C_{2k}-free graphs and of such C_{2k}-free bipartite graphs, where the number of vertices in the two classes are fixed.

Note that our question is strongly connected to the following problem: Erdš, Sárközy, T. Sós and Győri investigated how many numbers can be chosen from $\{1, 2, \ldots, n\}$ in such a way that the product of any $2k$ of them is not a perfect square. The maximal size of such a subset is denoted by $F_{2k}(n)$. The functions F and G clearly satisfy the inequality $F_{2k}(n) \leq G_k(n)$.

A problem in graph theory related to Poonen's conjecture
Shalom Eliahou (joint work with Youssef Fares)

Given a rational number c, consider the quadratic map $\varphi(x) = x^2 + c$ defined over the rationals. What are the possible cycle lengths under iteration of φ? Poonen conjectured that the answer is at most 3. In this work we obtain, by graph-theoretical means, an upper bound on cycle lengths which depends on the number k of prime factors of the denominator of c. We relate this to the graph problem consisting in determining the least number N_k of vertices such that every family of k simple graphs G on $n \geq N_k$ vertices and with stability number $\alpha(G) \leq 2$ must have a common edge. We show that N_k provides the above-mentioned upper bound on cycle lengths of φ.

Near universal cycles and ordered partitions of numbers
Michał Debski (joint work with Zbigniew Lonc)

A cyclic sequence of elements of $[n]$ is an (n, k)-Ucycle packing (respectively, (n, k)-Ucycle covering) if every k-subset of $[n]$ appears in this sequence at most once (resp. at least once) as a segment of consecutive terms. Let $p_{n,k}$ be the length of a longest (n, k)-Ucycle packing and $c_{n,k}$ the length of a shortest (n, k)-Ucycle covering.

We show, that there exist almost optimal Ucycle packings and coverings if k - as a function of n - does not grow too rapidly. For $k < n^{\frac{1}{3}}$ both $p_{n,k}$ and $c_{n,k}$ are equal to $\binom{n}{k} \pm o(\binom{n}{k}^{\beta})$ for some $\beta < 1$. If $k = o(n)$, then $p_{n,k} = \binom{n}{k} - o(\binom{n}{k})$.

In the proof of the latter result we use ordered partitions of numbers - where an ordered partition of n is a sequence of positive integers $n_1, \ldots, n_k$ that sum up to n (and reuse of symbols n and k is intentional). We show, that in almost every such partition the largest value of $n_1, \ldots, n_k$ appears only once (and the proof of this property will be sketched in the talk).

Strong edge-colouring of sparse planar graphs
Petru Valicov (joint work with Julien Bensmail, Ararat Harutyunyan, and Hervé Hocquard)

A strong edge-colouring of a graph is a proper edge-colouring where each colour class induces a matching. It is known that every planar graph with maximum degree Δ has a strong edge-colouring with at most $4\Delta + 4$ colours. We show that $3\Delta + 1$ colours suffice if the graph has girth 6, and observe that 4Δ colours suffice if $\Delta \geq 7$ or the girth is at least 5. We also raise some questions related to a long-standing conjecture of Vizing on proper edge-colouring of planar graphs.

The distance-t chromatic index of graphs
Tomáš Kaiser (joint work with Ross J. Kang)

A strong edge-colouring of a graph is one in which edges at distance at most two have distinct colours. Molloy and Reed proved that every graph of maximum degree Δ has a strong edge-colouring using $(2 - \epsilon)\Delta^2$ colours, where ϵ is a positive constant. We prove a corresponding result for the distance-t analogue of the strong chromatic index, showing that there is an absolute constant $\epsilon' > 0$ such that for all t, Δ and all graphs G of maximum degree Δ, the distance-t chromatic index of G is at most $(2 - \epsilon')\Delta^t$. We also consider the distance-t chromatic index of graphs with maximum degree Δ and girth at least $2t + 1$, for which we prove an upper bound of $O(\Delta^t / \log \Delta)$ as $\Delta \to \infty$.

S-Packing Colorings of Cubic Graphs
Nicolas Gastineau (joint work with Olivier Togni)

Given a non-decreasing sequence $S = (s_1, s_2, \ldots, s_k)$ of positive integers, an *S-packing coloring* of a graph G is a mapping c from $V(G)$ to $\{s_1, s_2, \ldots, s_k\}$ such that any two vertices with color s_i are at mutual distance greater than s_i, $1 \leq i \leq k$. We study S-packing colorings of (sub)cubic graphs and prove that they are $(1,2,2,2,2,2,2)$-packing colorable and $(1,1,2,2,3)$-packing colorable. For subdivisions of subcubic graphs we derive sharper bounds.

Uniquely packable trees
Elizabeth Jonck (joint work with Michael Dorfling)

An i-packing in a graph G is a set of vertices that are pairwise distance more than i apart. A *packing colouring* of G is a partition $X = \{X_1, X_2, \ldots, X_k\}$ of $V(G)$ such that each colour class X_i is an i-packing. The minimum order k of a packing colouring is called the packing chromatic number of G, denoted by $\chi_\rho(G)$. In this paper we investigate the existence of trees T for which there is only one packing colouring using $\chi_\rho(T)$ colours. For the case $\chi_\rho(T) = 3$, we completely characterise all such trees.

Identifying codes for families of split graphs
Annegret Wagler (joint work with Gabriela Argiroffo and Silvia Bianchi)

The identifying code problem is a newly emerging search problem, challenging both from a theoretical and a computational point of view, even for special graphs like bipartite graphs and split graphs. Hence, a typical line of attack for this problem is to determine minimum identifying codes of special graphs or to provide bounds for their size.

In this work we study minimum identifying codes for some families of split graphs: thin and thick headless spiders, complete suns and their complements. For that, we consider associated hypergraphs, discuss their combinatorial structure, and demonstrate how the related polyhedra can be entirely described or polyhedral arguments can be applied to find minimum identifying codes for special split graphs.

On Connected Identifying Codes for Infinite Lattices
Victor Campos (joint work with Fabricio S. Benevides, Mitre Dourado, Rudini Sampaio, and Ana Silva)

An *identifying code* in a graph G is a set C of vertices of G such that the closed neighbourhood of every vertex contains a unique and non-empty subset of C. We say that C is a *connected identifying code* if $G[C]$ is connected. We prove that if a finite graph G on n vertices has maximum degree Δ, then any connected identifying code C satisfies $|C| \geq \frac{2n-2}{\Delta+1}$. We also show this bound is best possible and that the coefficient of n cannot be improved for Δ-regular graphs. We also show that the minimum density of connected identifying codes for the infinite triangular, hexagonal and square lattices are $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{5}$, respectively.

Identifying codes in vertex-transitive graphs
Élise Vandomme (joint work with Sylvain Gravier, Aline Parreau, Sara Rottey, and Leo Storme)

We consider the problem of computing identifying codes of graphs and its fractional relaxation. The ratio between the optimal integer and fractional solutions is between 1 and $2\log(|V|)$ where V is the set of vertices of the graph. We focus on vertex-transitive graphs for which we can compute the exact fractional solution. There are known examples of vertex-transitive graphs that reach both bounds. We exhibit infinite families of vertex-transitive graphs with integer and fractional identifying codes of order $|V|^\alpha$ with $\alpha \in \{\frac{1}{4}, \frac{1}{3}, \frac{2}{5}\}$. These families are generalized quadrangles (strongly regular graphs based on finite geometries). They also provide examples for metric dimension of graphs.

Variations of identifying codes in graphs obtained by adding or removing one vertex

Olivier Hudry (joint work with Irène Charon, Iiro Honkala, and Antoine Lobstein)

Let G be a simple, undirected graph with vertex set V. For $v \in V$ and $r \geq 1$, we denote by $B_{G,r}(v)$ the ball of radius r and centre v. A set $\mathcal{C} \subseteq V$ is said to be an *r-identifying code* in G if the sets $B_{G,r}(v) \cap \mathcal{C}$, $v \in V$, are all nonempty and distinct. A graph G admitting an r-identifying code is called *r-twin-free*, and in this case the size of a smallest r-identifying code in G is denoted by $\gamma_r(G)$.

We study the following structural problem: let G be an r-twin-free graph, and G_v be a graph obtained from G by adding or deleting a vertex v. If G_v is still r-twin-free, we compare the behaviours of $\gamma_r(G)$ and $\gamma_r(G_v)$, establishing results on their possible differences and ratios.

1-factor and cycle covers of cubic graphs

Eckhard Steffen)

Let G be a bridgeless cubic graph. Consider a list of k 1-factors of G. Let E_i be the set of edges contained in precisely i members of the k 1-factors. Let $\mu_k(G)$ be the smallest $|E_0|$ over all lists of k 1-factors of G.

Any list of three 1-factors induces a core of a cubic graph. We use results on the structure of cores to prove sufficient conditions for Berge-covers and for the existence of three 1-factors with empty intersection. Furthermore, if $\mu_3(G) \neq 0$, then $2\mu_3(G)$ is an upper bound for the girth of G. We also prove some new upper bounds for the length of shortest cycle covers of bridgeless cubic graphs.

Cubic graphs with $\mu_4(G) = 0$ have a 4-cycle cover of length $\frac{4}{3}|E(G)|$ and a 5-cycle double cover. These graphs also satisfy two conjectures of Zhang. We also give a negative answer to a problem of C.-Q. Zhang.

Directed cycle double cover conjecture: fork graphs
Andrea Jiménez (joint work with Martin Loebl)

We explore the well-known Jaeger's directed cycle double cover conjecture which is equivalent to the assertion that every cubic bridgeless graph has an embedding on a closed Riemann surface without dual loops. We define a new class of graphs that we call *lean-fork graphs.* Fork graphs are cubic bridgeless graphs that admit ear decompositions, which start from a triangle and connect fairly short ears; in addition, Y$-\Delta$, $\Delta-$Y transformations are allowed. Lean-fork graphs are fork graphs that fulfill a connectivity property. We establish that Jaeger's conjecture is valid for the class of lean-fork graphs. Moreover, we show that for each cubic bridgeless graph G there exists a lean-fork graph that contains a subdivision of G as an induced subgraph. Our results establish for the first time, to the best of our knowledge, the validity of Jaeger's conjecture in a rich inductively defined class of graphs.

Decomposing integer flows in signed graphs into characteristic flows
Edita Máčajová (joint work with Martin Škoviera)

We generalise to signed graphs a result of Tutte [Canad. J. Math. 8 (1956), 13–28] concerning decomposition of an integer flow into a sum of elementary flows. As an application we show that a signed graph G admitting a nowhere-zero k-flow has a signed circuit cover of length at most $2(k-1)|E(G)|$.

Nowhere-zero flows on signed series-parallel graphs
Edita Rollová (joint work with Tomáš Kaiser)

Bouchet conjectured in 1983 that each signed graph which admits a nowhere-zero flow has a nowhere-zero 6-flow. We prove that the conjecture is true for all signed series-parallel graphs. Unlike the unsigned case, the restriction to series-parallel graphs is nontrivial; in fact, the result is tight for infinitely many graphs.

A domination algorithm for $\{0,1\}$-instances of the travelling salesman problem
Viresh Patel (joint work with Daniela Kühn, Deryk Osthus)

We present an approximation algorithm for $\{0,1\}$-instances of the travelling salesman problem which performs well with respect to combinatorial dominance. More precisely, we give a polynomial-time algorithm which has domination ratio $1 - n^{-1/29}$. In other words, given a $\{0,1\}$-edge-weighting of the complete graph K_n on n vertices, our algorithm outputs a Hamilton cycle H^* of K_n with the following property: the proportion of Hamilton cycles of K_n whose weight is smaller than that of H^* is at most $n^{-1/29}$. Our analysis is based on a martingale approach. Previously, the best result in this direction was a polynomial-time algorithm with domination ratio $1/2-o(1)$ for arbitrary edge-weights. We also prove a hardness result showing that, if the Exponential Time Hypothesis holds, there exists a constant C such that $n^{-1/29}$ cannot be replaced by $\exp(-(\log n)^C)$ in the result above.

A $\frac{5}{4}$-approximation for subcubic 2EC using circulations
Sylvia Boyd (joint work with Yao Fu and Yu Sun)

In this presentation we study the NP-hard problem of finding a minimum size 2-edge-connected spanning subgraph (henceforth 2EC) in a given subcubic multigraph. For such graphs we present a new $\frac{5}{4}$-approximation algorithm, improving upon the current best approximation ratio of $\frac{5}{4} + \varepsilon$ both in ratio and simplicity of proof. Our algorithm involves an elegant new method based on circulations which we feel has the potential to be more broadly applied.

A New Formulation of Degree-Constrained Spanning Problems

Miklós Molnár (joint work with Massinissa Merabet and Sylvain Durand)

Given a graph with edge-costs, searching for a minimum cost structure that connects a subset of vertices is a classic problem. We examine the spanning problems under constraints on the vertex degrees. Spanning tree solutions were generally investigated to solve them. However, for some applications the solution is not necessarily a sub-graph. Assuming that the degree constraint is due to the limited instantaneous capacity of the vertex and that the only other constraint on the spanning structure is its connectivity, we propose a reformulation of some spanning problems. To find the optimal coverage of the concerned vertices, an extension of the tree concept has been proposed. A hierarchy is obtained by a graph homomorphism between a tree and a target graph. Since this spanning structure may refer vertices (and edges) of the target graph several times, it is more flexible to satisfy constraints and nevertheless pertinent for network applications. Hierarchies correspond to the optimal solutions of the new problems. Here we resume our first promising results on the degree-constrained spanning hierarchies. They can solve network related cases where trees meeting the constraints do not exist. In other cases, hierarchies outperform trees. Furthermore, the degree constrained spanning hierarchy problem can be approximated within a constant ratio (while it is not possible with trees).

Properties of Graph ATSP
Corinna Gottschalk

We consider the Graph Asymmetric Traveling Salesman Problem and present a family of instances that give a lower bound of $\frac{3}{2}$ for the integrality ratio of the Held-Karp bound for Graph ATSP. In contrast to the families of examples that provide a lower bound of 2 for the integrality ratio of ATSP, we only use a number of nodes linear in $\frac{1}{\varepsilon}$ to achieve a ratio of $\frac{3}{2} - \varepsilon$. Furthermore, we also show how the properties of Graph ATSP might be helpful to improve the approximation ratio for this problem, since it suffices to find a generalization of a thin tree.

Multicolor Ramsey Numbers for long cycles versus some sequences of disjoint paths
Halina Bielak (joint work with Kinga Dąbrowska)

We present the multicolor Ramsey numbers for some sequences of disjoint unions of graphs. We count $R(G_1, G_2, \cdots, G_k, C_m)$ where G_i $(1 \leq i \leq k)$ is a disjoint union of some paths. We generalize results of Faudree and Schelp, Bielak, Dzido, Shiu et. al., and Omidi and Raeisi.

Edge-colorings avoiding a fixed matching with a prescribed color pattern
Carlos Hoppen (joint work with Hanno Lefmann)

We consider an extremal problem motivated by a question of Erdős and Rothschild regarding edge-colorings of graphs avoiding a given monochromatic subgraph, which was later extended by Balogh to edge-colorings avoiding subgraphs with a fixed coloring. Given a natural number r and a graph F, an r-pattern P of F is a partition of the edge set of F into r (possibly empty) classes, and an r-coloring of the edge set of a graph G is said to be (F, P)-free if it does not contain a copy of F in which the partition of the edge set induced by the coloring coincides with P. Let $c_{r,(F,P)}(G)$ be the number of (F,P)-free r-colorings of a graph G. For large n, we maximize this number over all n-vertex graphs for a large class of patterns in matchings and we describe the graphs that achieve this maximum.

Path-kipas Ramsey numbers

Binlong Li (joint work with Halina Bielak and Přemysl Holub)

Let G_1 and G_2 be two given graphs. The Ramsey number $R(G_1, G_2)$ is the least integer r such that for every graph G on r vertices, either G contains a G_1 or $\overline{G}$ contains a G_2. We use P_n to denote the path on n vertices, and $\widehat{K}_m$ the kipas on $m+1$ vertices, i.e., the graph obtained by joining K_1 and P_m. In this paper, we determined the exact value of the path-kipas Ramsey numbers $R(P_n, \widehat{K}_m)$ for all n, m.

A precise threshold for quasi-Ramsey numbers

Guus Regts (joint work with Ross J. Kang, János Pach, and Viresh Patel)

We consider a variation of Ramsey numbers introduced by Erdős and Pach, where instead of seeking complete or independent sets we only seek a *t-homogeneous set*, a vertex subset that induces a subgraph of minimum degree at least t or the complement of such a graph.

For any $\nu > 0$ and positive integer k, we show that any graph G or its complement contains as an induced subgraph some graph H on $\ell \geq k$ vertices with minimum degree at least $\frac{1}{2}(\ell - 1) + \nu$ provided that G has at least $k^{\Omega(\nu^2)}$ vertices. We also show this to be best possible in a sense. This may be viewed as correction to a result claimed by Erdős and Pach.

For the above result, we permit H to have order at least k. In the harder problem where we insist that H have exactly k vertices, we do not obtain sharp results, although we show a way to translate results of one form of the problem to the other.

On-Line Choice Number of Complete Multipartite Graphs
Hong-Bin Chen (joint work with Fei-Huang Chang, Jun-Yi Guo, and Yu-Pei Huang)

This paper studies the on-line choice number on complete multipartite graphs with independence number m. We give a unified strategy for every prescribed m. Our main result leads to several interesting consequences comparable to known results. (1) If $k_1 - \sum_{p=2}^{m} \left(\frac{p^2}{2} - \frac{3p}{2} + 1\right) k_p \geq 0$, where k_p denotes the number of parts of cardinality p, then G is on-line chromatic-choosable. (2) If $|V(G)| \leq \frac{m^2-m+2}{m^2-3m+4}\chi(G)$, then G is on-line chromatic-choosable. (3) The on-line choice number of regular complete multipartite graphs $K_{m\star k}$ is at most $\left(m + \frac{1}{2} - \sqrt{2m-2}\right) k$ for $m \geq 3$.

Partial list colouring of certain graphs
Rogers Mathew (joint work with Jeannette Janssen and Deepak Rajendraprasad)

Let G be a graph on n vertices and let $\mathcal{L}_k$ be an arbitrary function that assigns each vertex in G a list of k colours. Then G is *$\mathcal{L}_k$-list colourable* if there exists a proper colouring of the vertices of G such that every vertex is coloured with a colour from its own list. We say G is *k-choosable* if for every such function $\mathcal{L}_k$, G is $\mathcal{L}_k$-list colourable. The minimum k such that G is k-choosable is called the *list chromatic number* of G and is denoted by $\chi_L(G)$. Let $\chi_L(G) = s$ and let t be a positive integer less than s. The *partial list colouring conjecture* due to Albertson et al. states that for every $\mathcal{L}_t$ that maps the vertices of G to t-sized lists, there always exists an induced subgraph of G of size at least $\frac{tn}{s}$ that is $\mathcal{L}_t$-list colourable. In this paper we show that the partial list colouring conjecture holds true for certain classes of graphs like claw-free graphs, graphs with large chromatic number, chordless graphs, and series-parallel graphs.

In the second part of the paper, we put forth a question which is a variant of the partial list colouring conjecture: does G always contain an induced subgraph of size at least $\frac{tn}{s}$ that is t-choosable? We show that the answer to this question is not always 'yes' by explicitly constructing an infinite family of connected 3-choosable graphs where a largest induced 2-choosable subgraph of each graph in the family is of size at most $\frac{5n}{8}$.

On total $L(2,1)$-coloring regular grids and diameter two graphs

Daniel F. D. Posner (joint work with Márcia R. Cerioli)

A *total $L(2,1)$-coloring* of a graph $G = (V, E)$ is a function $f : (V \cup E) \to \mathbb{N}$ such that if $uv \in E$, then $|f(u) - f(v)| \geq 2$, $|f(uv) - f(u)| \geq 2$, and $|f(uv) - f(v)| \geq 2$; moreover, if $uv, vw \in E$, then $f(uv) \neq f(vw)$, $f(u) \neq f(vw)$, $f(w) \neq f(uv)$, and $f(u) \neq f(w)$. The maximum label used in f is its *span* and the minimum span among all total $L(2,1)$-colorings of a graph G is denoted by $\lambda_T(G)$. We show how to efficiently obtain a total $L(2,1)$-coloring with span λ_T for regular grids graphs. Moreover, we state an analogous of the known Griggs and Yeh's conjecture for total $L(2,1)$-colorings and prove it for diameter two graphs. We also establish tight lower and upper bounds for λ_T on cographs and threshold graphs, two subclasses of diameter two graphs.

On equitable total coloring of cubic graphs

Diana Sasaki (joint work with Simone Dantas, Celina M.H. de Figueiredo, Myriam Preissmann, and Vinícius F. dos Santos)

A total coloring is equitable if the number of elements colored with each color differs by at most one. The least k for which G has such a coloring is the equitable total chromatic number of G, denoted by $\chi''_e(G)$. It is conjectured that $\chi''_e(G) \leq \Delta + 2$ for any graph G, and this conjecture was proved for cubic graphs. In this work, we investigate the equitable total coloring of cubic graphs. We prove that the problem of determining if a cubic bipartite graph has an equitable 4-total-coloring is NP-complete and we determine that the equitable total chromatic number of all members of the Goldberg snark family is 4.

On the strongest form of a theorem of Whitney for hamiltonian cycles in plane triangulations

Nico Van Cleemput (joint work with Gunnar Brinkmann and Jasper Souffriau)

We investigate hamiltonian cycles in triangulations. The central part of the talk is the search for the strongest possible form of Whitney s theorem about hamiltonian triangulations in terms of the decomposition tree defined by separating triangles. Jackson and Yu showed that a triangulation is hamiltonian if this decomposition tree has maximum degree 3. We will decide on the existence of non-hamiltonian triangulations with given decomposition trees for all trees except trees with exactly one vertex with degree $k \in \{4, 5\}$ and all other degrees at most 3. For these cases we show that it is sufficient to decide on the existence of non-hamiltonian triangulations with decomposition tree $K_{1,4}$ or $K_{1,5}$, and we give several restrictions on the structure of such non-hamiltonian triangulations. These results were obtained using a combination of computational results and theoretical results, and both will be explained.

Hamiltonian Cycles in k-Connected k-Regular Graphs

Letícia R. Bueno (joint work with Jorge L.B. Pucohuaranga and Simone Dantas)

A long-standing conjecture states that every 4-connected 4-regular claw-free graph is hamiltonian. These graphs fall into three classes

$\mathcal{G}_0$, $\mathcal{G}_1$ and $\mathcal{G}_2$, of which only $\mathcal{G}_1$ is known to be hamiltonian. In this paper, we prove that $\mathcal{G}_0$ is hamiltonian and that $\mathcal{G}_2$ is prism-hamiltonian, also corroborating to another conjecture that the prism over every 4-connected 4-regular graph is hamiltonian. Also, we prove that the prism over every k-connected k-regular bipartite graph is hamiltonian, for $k \geq 2$.

Hamiltonian cycles in spanning subgraphs of line graphs
Hao Li (joint work with Weihua He, Weihua Yang, and Yandong Bai)

Let G be a graph and $e = uv$ an edge in G (also a vertex in the line graph $L(G)$ of G). Then e is in two cliques $E_G(u)$ and $E_G(v)$ with $E_G(u) \cap E_G(v) = \{e\}$ of $L(G)$, that correspond to all edges incident with u and v in G respectively. Let $SL(G)$ be any spanning subgraph of $L(G)$ such that every vertex $e = uv$ is adjacent to at least $\min\left(d_G(u) - 1, \lceil \frac{3}{4} d_G(u) + \frac{1}{2} \rceil\right)$ vertices of $E_G(u)$ and to at least $\min\left(d_G(v) - 1, \lceil \frac{3}{4} d_G(v) + \frac{1}{2} \rceil\right)$ vertices of $E_G(v)$. Then if $L(G)$ is hamiltonian, we show that $SL(G)$ is hamiltonian. As a corollary we obtain a lower bound on the number of edge-disjoint hamiltonian cycles in $L(G)$.

Hamiltonian cycles in generalizations of bipartite tournaments
Ilan A. Goldfeder (joint work with Hortensia Galeana-Sánchez)

The existence of Hamiltonian cycles in bipartites was characterized by Gregory Gutin, Roland Häggkvist and Yannis Manoussakis. Later, Jørgen Bang-Jensen introduced some generalizations of bipartites tournaments, namely $\mathcal{H}_i$-free digraphs, for i in $\{1, 2, 3, 4\}$. Bang-Jensen conjectured that an $\mathcal{H}_i$-free digraph D, for i in $\{1, 2, 3, 4\}$, is Hamiltonian if and only if D is strong and contains a cycle factor (that is, a collection of vertex disjoint cycles covering all the vertices of D). Particularly, a digraph D is $\mathcal{H}_4$-free if for every four distinct vertices u, v, w, x in D such that $u \to v \leftarrow w \to x$, the vertices u and x are adjacent in D. Shiying Wang and Ruixia Wang proved the conjecture for i in $\{1, 2\}$ in 2009 and Hortensia Galeana-Sánchez, Ilan A. Goldfeder and Isabel Urrutia proved the conjecture for $i = 3$ in 2010. In this talk, we will discuss the case $i = 4$.

Poset Entropy versus Number of Linear Extensions: the Width-2 Case

Selim Rexhep (joint work with Samuel Fiorini)

Kahn and Kim (J. Comput. Sci., 1995) have shown that for a finite poset P, the entropy of the incomparability graph of P (normalized by multiplying by the order of P) and the base-2 logarithm of the number of linear extensions of P are within constant factors from each other. The tight constant for the upper bound was recently shown to be 2 by Cardinal, Fiorini, Joret, Jungers and Munro (STOC 2010, Combinatorica). Here, we refine this last result in case P has width 2: we show that the constant can be replaced by $2 - \varepsilon$ if one also takes into account the number of connected components of size 2 in the incomparability graph of P. Our result leads to a better upper bound for the number of comparisons in algorithms for the problem of sorting under partial information.

An Enumeration of Distance-Hereditary and 3-Leaf Power Graphs

Cédric Chauve (joint work with Éric Fusy and Jérémie Lumbroso)

Distance-hereditary graphs form an important class of graphs, due to the fact that they are the graphs which are totally decomposable graphs under the split decomposition. Nakano et al. (J. Comp. Sci. Tech., 2007) constructively proved that the number of distance-hereditary graphs on n vertices is bounded by $2^{\lceil 3.59n \rceil}$. In the present work we refine this result and provide experimental evidence that the number of unlabelled distance-hereditary graphs on n vertices is bounded by 2^{3n}. We also provide evidence that the number of unlabelled 3-leaf power graphs, a sub-class of distance-hereditary graphs of interest in phylogenetics, on n vertices is bounded by 2^{2n}.

The Fibonacci numbers of certain subgraphs of Circulant graphs

Loiret Alejandría Dosal-Trujillo (joint work with Hortensia Galeana-Sánchez)

The Fibonacci number of a graph is the total number of its independent vertex sets. In general, the problem to find the Fibonacci number of a graph is NP-complete. Prodinger and Tichy proved that the Fibonacci number of the path of order n is the $n+2$-Fibonacci number; and the Fibonacci number of the cycle of order n is the n-Lucas number. A circulant graph $C_{n(m_1,m_2,\ldots,m_r)}$ is a graph with vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and edge set $E = \{v_i v_{i+m_j \pmod{n}} : i \in \{1, 2, \ldots, n\}, j \in \{1, 2, \ldots, r\}\}$, where $r \in \mathbb{Z}^+$. The values m_j are the jump sizes. In this talk we will discuss about the Fibonacci numbers of the circulant graphs of order n with r consecutive jumps $1, 2, \ldots, r$ and of several subgraphs of this family. We will see that these numbers are completely determined by some sequences that generalize the Fibonacci and Lucas sequences.

The number of labeled connected graphs modulo odd integers

Arun P. Mani (joint work with Rebecca J. Stones)

Let $c(n)$ denote the number of labeled connected graphs on n vertices. Whenever $n > 0$, we prove the linear recurrence congruence $c(n+m) \equiv 2^{\varphi(m)/2} c(n+m-\varphi(m)) \pmod{m}$ for every odd integer $m > 1$, where $\varphi(m)$ is the Euler totient function, and thus show that the sequence $(c(n) : n \in \mathbb{Z}_{>0})$ is ultimately periodic modulo every odd integer $m > 1$ with period either $\varphi(m)$ or $2\varphi(m)$.

On vertices of the Boolean quadric polytope relaxations
Andrei Nikolaev

We study the relaxations $M_{n,k}$ of the Boolean quadric polytope BQP_n, obtained by imposing a sequence of linear constraints defining BQP_k. This sequence of polytopes preserves the integral vertices of the BQP_n, with every relaxation having its own unique fractional vertices. The first two relaxations M_n and $M_{n,3}$ are in one-to-one correspondence via the covariance mapping with the rooted semimetric and metric relaxations of the cut polytope. The following questions are considered: denominators of the $M_{n,k}$ fractional vertices, shared fractional vertices of different relaxations, and the complexity of the integrality recognition problem on $M_{n,k}$ (for a given linear objective function $f(x)$ and a polytope M determine whether $\max\{f(x) \mid x \in M\}$ achieved at an integral vertex of M).

An extension of Lehman's theorem and ideal set functions
Tamás Király (joint work with Júlia Pap)

Lehman's theorem on the structure of minimally nonideal clutters is one of the fundamental results of polyhedral combinatorics. One approach to extend it has been to give a common generalization with the characterization of minimally imperfect clutters. We give a new generalization of this kind, which combines two types of covering inequalities and works well with the natural definition of minors. We also show how to extend the notion of idealness to unit-increasing set functions, in a way that is compatible with minors and blocking operations.

The Trader Multiflow problem: When the cut cone is box-TDI
Roland Grappe (joint work with Denis Cornaz and Mathieu Lacroix)

The circuit inequalities of a graph form, together with the nonnegativity constraints, a linear system describing the cut cone of the graph, which is TDI if and only if the latter is series-parallel. We prove that this system is actually box-TDI. As a consequence, the Trader Multiflow problem, which is a generalization of the Multiflow problem where one may buy more capacity and sell additional flow, is polynomial when the graph is series-parallel.

Vertex Shelling Polytopes of Split Graphs
Keno Merckx

Korte and Lovász provided several results about the characterization of antimatroids polytopes. We have managed to extend to new cases their work, our main contribution is to obtain the complete linear description of the polytope related to antimatroids built on split graphs.

A bound on the number of edges in graphs without an even cycle
Boris Bukh (joint work with Zilin Jiang)

We show that, for each fixed k, an n-vertex graph not containing a cycle of length $2k$ has at most $80\sqrt{k \log k} \cdot n^{1+1/k} + O(n)$ edges.

Supersaturation Problem for Color-Critical Graphs
Zelealem B. Yilma (joint work with Oleg Pikhurko)

The *Turán function* $\mathrm{ex}(n, F)$ of a graph F is the maximum number of edges in an F-free graph with n vertices. The classical results of Turán and Rademacher from 1941 led to the study of supersaturated graphs where the key question is to determine $h_F(n, q)$, the minimum number of copies of F that a graph with n vertices and $\mathrm{ex}(n, F) + q$ edges can have.

We determine $h_F(n, q)$ asymptotically when F is *color-critical* (that is, F contains an edge whose deletion reduces its chromatic number) and $q = o(n^2)$.

Determining the exact value of $h_F(n, q)$ seems rather difficult. For example, let c_1 be the limit superior of q/n for which the extremal structures are obtained by adding some q edges to a maximal F-free graph. The problem of determining c_1 for cliques was a well-known question of Erdős that was solved only decades later by Lovász and Simonovits. Here we prove that $c_1 > 0$ for every color-critical F. Our approach also allows us to determine c_1 for a number of graphs, including odd cycles, cliques with one edge removed, and complete bipartite graphs plus an edge.

Decompositions of graphs into induced subgraphs
Zsolt Tuza (joint work with Csilla Bujtás)

We consider an extremal graph problem concerning edge decompositions, raised by Bondy and Szwarcfiter. Given a graph F, an induced F-decomposition of G is a collection of induced subgraphs $F_i \subset G$ which are mutually edge-disjoint, each of them is isomorphic to F, and $\bigcup_i E(F_i) = E(G)$. The problem is to determine the maximum number $\mathrm{ex}^*(n, F)$ of edges in a graph of order n which admits an induced F-decomposition. Our main result yields a characterization of graphs F such that $\binom{n}{2} - \mathrm{ex}^*(n, F) = \Theta(n)$ as $n \to \infty$.

Making a C_6-free graph C_4-free and bipartite
Ervin Győri (joint work with Scott Kensell and Casey Tompkins)

We show that every C_6-free graph G has a C_4-free, bipartite subgraph with at least $3e(G)/8$ edges. Our proof uses probabilistic and deterministic tools and a theorem of Füredi, Naor and Verstraete on C_6-free graphs.

An extension of Richardson's theorem in m-colored digraphs
Hortensia Galeana-Sánchez (joint work with Rocío Sánchez-López)

In this talk, we will show the existence of kernels by monochromatic paths in arc-colored digraphs. In particular, we will prove a generalization of Richardson's Theorem: Let D be an m-coloured digraph and $\mathscr{C}_C(D)$ its color-class digraph. If $\mathscr{C}_C(D)$ has no cycles of odd length at least 3, then D has a kernel by monochromatic paths, where the vertices of the color-class digraph $\mathscr{C}_C(D)$ are the colors represented in the arcs of D, and $(i,j) \in A(\mathscr{C}_C(D))$ if and only if there exist two arcs namely (u,v) and (v,w) in D such that (u,v) has color i and (v,w) has color j.

On panchromatic digraphs and the panchromatic number
Ricardo Strausz (joint work with Hortensia Galeana-Sánchez)

Let $D = (V, A)$ be a simple finite digraph, and let $\pi(D)$, the *panchromatic number* of D, be the maximum number of colours k such that for each (effective, or onto) colouring of its arcs $\varsigma\colon A \to [k]$ a *monochromatic path kernel* $N \subset V$ exists. It is not hard to see that D has a *kernel* – in the sense of Von Neumann – if and only if $\pi(D) = |A|$. In this note this invariant is introduced and some of its structural bounds are studied. For example, the celebrated theorem of Sands et al., in terms of this invariant, settles that $\pi(D) \geq 2$. It will be proved that

$$\pi(D) < |A| \iff \pi(D) < \min\left\{2\sqrt{\chi(D)}, \chi(L(D)), \theta(D) + \max \mathrm{d}_c(K_i) + 1\right\},$$

where $\chi(\cdot)$ denotes the chromatic number, $L(\cdot)$ denotes the line digraph, $\theta(\cdot)$ denotes the minimum partition into complete graphs of the underlying graph and $\mathrm{d}_c(\cdot)$ denotes the dichromatic number. We also introduce the notion of a *panchromatic* digraph which is a digraph D such that for every $k \leq |A|$ and every k-colouring of its arcs, it has a monochromatic path kernel. Some classes of panchromatic digraphs are further characterised.

On sums of graph eigenvalues
Evans M. Harrell (joint work with Joachim Stubbe)

We use two variational techniques to prove upper bounds for sums of the lowest several eigenvalues of matrices associated with finite, simple, combinatorial graphs. These include estimates for the adjacency matrix of a graph and for both the standard combinatorial Laplacian and the renormalized Laplacian. We also provide upper bounds for sums of squares of eigenvalues of these three matrices. Among our results, we generalize an inequality of Fiedler for the extreme eigenvalues of the graph Laplacian to a bound on the sums of the smallest (or largest) k such eigenvalues, $k < n$.

On solutions of several conjectures about remoteness and proximity in graphs
Jelena Sedlar

Remoteness and proximity are recently introduced graph invariants, remoteness being the maximum averaged sum of distances from a vertex to all others, while the proximity is the minimum of such sums. Several conjectures were posed by Aouchiche and Hansen in 2011 which involve remoteness and proximity of a graph. We prove the conjecture about the upper bound for the difference of average distance and proximity in a graph. The other two conjectures, one involving the difference of average eccentricity and remoteness and the other involving the difference of remoteness and radius, we prove for trees.

Gyárfás conjecture is almost always true
Yelena Yuditsky (joint work with Bruce Reed)

We study the structure of typical T-free graphs, i.e. graphs which do not contain a specific tree T as an induced subgraph. One way of showing that G does not contain a specific graph H as an induced subgraph is to provide a partition $S_1, S_2, ..., S_t$ of $V(G)$ s.t. for any partition $X_1, X_2, ..., X_t$ of $V(H)$, there is some i such that the subgraph of H induced by X_i is not an induced subgraph of the subgraph of G induced by S_i. We call such a partition a *witnessing* *partition.* Trivially, any H-free graph permits a witnessing partition with $t = 1$. Witnessing partitions into $\tau(H) = \max_t\{\exists a + b = t | H$ can not be partitioned into a stable sets and b cliques$\}$ are of a particular interest. We show that for every tree T and G s.t. $T \not\subseteq_I G$ there is an $\epsilon(T) > 0$ and an exceptional set S of $|V(G)|^{1-\epsilon(T)}$ vertices, and a witnessing partition of $G \setminus S$ into $\tau(T)$ parts. This allows us to show that for all T almost every T-free G satisfies $\chi(G) = (1+o(1))\omega(G)$. For most trees T and almost all T-free G, we can actually find such a witnessing partition of all $V(G)$ s.t. there is some S_i which is a clique and it allows us to show $\chi(G) = \omega(G)$.

New Upper Bounds for the Acyclic Chromatic Index
Anton Bernshteyn

An edge coloring of a graph G is called an *acyclic edge coloring* if it is proper (i.e. adjacent edges receive different colors) and every cycle in G contains edges of at least three different colors (there are no *bichromatic cycles* in G). The least number of colors needed for an acyclic edge coloring of G is called the *acyclic chromatic index* of G and is denoted by $a'(G)$. It is conjectured by Fiamčik and independently by Alon *et al.* that $a'(G) \leq \Delta(G) + 2$, where $\Delta(G)$ denotes the maximum degree of G. However, the best known general bound is $a'(G) \leq 4(\Delta(G) - 1)$ due to Esperet and Parreau, which was obtained using the *entropy compression method.* We apply this method to show that if G contains no 4-cycle, then $a'(G) \leq 3\Delta(G) + o(\Delta(G))$. Moreover, for every $\varepsilon > 0$ there exists a constant c such that if $g(G) \geq c$, then $a'(G) \leq (2 + \varepsilon)\Delta(G) + o(\Delta(G))$, where $g(G)$ denotes the girth of G.

Entropy compression method applied to graph colorings
Alexandre Pinlou (joint work with Daniel Gonçalves and Mickaël Montassier)

We propose a framework based on the entropy compression method, inspired by the one of Esperet and Parreau, to prove upper bounds for some chromatic numbers. From this method, in particular, we derive that every graph with maximum degree Δ has an acyclic vertex-coloring using at most $\frac{3}{2}\Delta^{\frac{4}{3}} + O(\Delta)$ colors, and a non-repetitive vertex-coloring using at most $\Delta^2 + 1.89\Delta^{\frac{5}{3}} + O(\Delta^{\frac{4}{3}})$ colors.

Harmonious Coloring of Hypergraphs
Sebastian Czerwiński (joint work with Bartłomiej Bosek, Jarosław Grytczuk, and Paweł Rzażewski)

In this paper we introduce and study several variants of *harmonious* coloring of hypergraphs. Given a hypergraph H the problem is to color its vertices so that no two edges get the same color pattern (a set, a multiset, a sequence of colors, etc.). We prove a few results showing linear dependence of the related chromatic parameters on the least possible value following from the number of different patterns available. We focus on uniform hypergraphs, but we also consider planar hypergraphs and shift hypergraphs. The methods we use are basically probabilistic, but we also explore a novel and powerful entropy compression arguments.

Approximation Algorithm for the Fault Tolerant Virtual Backbone in a Wireless Sensor Network
Zhao Zhang (joint work with Jiao Zhou, Yishuo Shi, and Yaping Zhang)

A subset of nodes $C \subseteq V$ is a (k, m)-CDS if every node in $V \setminus C$ is adjacent to at least m nodes in C and the subgraph of G induced by C is k-connected. In this talk, I shall introduce current state of art studies on (k, m)-CDS and present two of our recent works in this aspect. The first one computes a $(1, m)$-CDS with performance ratio at most $2 + \ln(\Delta + m - 2)$, where Δ is the maximum degree of the graph. The second one computes a $(2, m)$-CDS ($m \geq 2$) with performance ratio $\alpha + 2(1 + \ln \alpha)$, where α is the approximation ratio for the minimum $(1, m)$-CDS problem. These results improve on previous approximation ratios for the problem.

The complexity of finding arc-disjoint branching flows
Jørgen Bang-Jensen (joint work with Frédéric Havet and Anders Yeo)

The concept of arc-disjoint flows in networks was recently introduced by Bang-Jensen and Bessy. This is a very general framework in which many well-known and important problems can be formulated. In particular, the existence of arc-disjoint branching flows, that is, flows which send one unit of flow from a given source s to all other vertices, generalizes the concept of arc-disjoint out-branchings (spanning out-trees) in a digraph. A pair of out-branchings $B^+_{s,1}, B^+_{s,2}$ from a root s in a digraph $D = (V, A)$ on n vertices corresponds to arc-disjoint branching flows x_1, x_2 (the arcs carrying flow in x_i are those used in $B^+_{s,i}$, $i = 1, 2$) in the network that we obtain from D by giving all arcs capacity $n - 1$. Using any maximum flow algorithm we can decide in polynomial time whether a given network $\mathcal{N} = (V, A, u)$ (u is the capacity function on the arcs of $\mathcal{N}$) has a branching flow x from a given root s such that $x_{ij} \leq u_{ij}$ for all $ij \in A$. It is then a natural question to ask how much we can lower the capacities on the arcs and still have, say, two arc-disjoint branching flows from the given root s. We prove that for every fixed integer $k \geq 2$ it is

- an NP-complete problem to decide whether a network $\mathcal{N} = (V, A, u)$ where $u_{ij} = k$ for every arc ij has two arc-disjoint branching flows.
- a polynomial problem to decide whether a network $\mathcal{N} = (V, A, u)$ on n vertices and $u_{ij} = n - k$ for every arc ij has two arc-disjoint branching flows.

The algorithm for the later result generalizes the polynomial algorithm due to Lovász for deciding whether a given input digraph has two arc-disjoint out-branchings rooted at a given vertex.

Augmenting graphs to become (k,ℓ)-redundant
Csaba Király

A graph $G=(V,E)$ is called (k,ℓ)-tight if $i(X) \leq k|X|-\ell$ for all $X \subseteq V$ with $|X| \geq 2$ and $|E| = k|X|-\ell$. A graph G is called (k,ℓ)-redundant if after the omission of any edge of G, $G-e$ still has a spanning (k,ℓ)-tight subgraph. In rigidity theory, (k,ℓ)-tight graphs play an important role as the minimal (generic) rigidity of a graph is equivalent with its (k,ℓ)-tightness for some $k \in \mathbb{Z}_+$ and $\ell \in \mathbb{Z}$ in some rigidity classes. Thus an algorithm for augmenting a graph to a (k,ℓ)-redundant graph can be used to augment the rigidity of this graph.

Here, we give a polynomial algorithm for the case where we want to augment a (k,ℓ)-tight graph when $\ell \leq \frac{3}{2}k$. We also sketch a polynomial algorithm for the general augmentation problem for $k \geq \ell$.

Topological Conditions for Reliable Broadcast in *Ad Hoc* Networks
Aris Pagourtzis (joint work with Chris Litsas, Giorgos Panagiotakos, and Dimitris Sakavalas)

We study the Reliable Broadcast problem in incomplete networks, under the *locally bounded adversarial model*, that is, there is a known bound on the number of players that a Byzantine adversary controls in each player's neighborhood. We review results which provide an exact characterization of the class of graphs in which Koo's Certified Propagation Algorithm (CPA) can achieve Reliable Broadcast. This approach allows us to settle an open question of Pelc and Peleg in the affirmative, by showing that CPA for *ad hoc* networks is indeed *unique*, that is, it can tolerate as many local corruptions as any other non-faulty algorithm, thus having optimal resilience. On the other hand, we show that it is NP-hard to check whether this condition holds for a given graph G.

Destroying Longest Cycles in Graphs
Susan A. van Aardt (joint work with Alewyn Burger, Jean E. Dunbar, Marietjie Frick, Bernardo Llano, Carsten Thomassen, and Rita Zuazua)

In 1976, C. Thomassen proved that in any graph one can destroy all the longest cycles by deleting at most one third of the vertices. We show that for graphs with circumference k, where k is at most 8, it suffices to remove at most one k'th of the vertices. The Petersen graph demonstrates that this result cannot be extended to include $k = 9$ but we show that in every graph with circumference nine we can destroy all 9-cycles by removing one fifth of the vertices.

Destroying Longest Cycles in Digraphs
Marietjie Frick (joint work with Susan A. van Aardt, Alewyn Burger, Jean E. Dunbar, Bernardo Llano, Carsten Thomassen, and Rita Zuazua)

The length of a longest cycle in a digraph D is called the circumference of D. We show that in any digraph with circumference k one can destroy all cycles by deleting a solely k-dependent fraction of the vertices. We also show that in a digraph with circumference k one can destroy all the longest cycles by deleting one k'th of the vertices if k equals 2 or 3, but this does not hold for any k bigger than 3.

On a reduction of 3-path Vertex Cover Problem to Vertex Cover Problem
Christoph Brause

The Vertex Cover Problem, also known as 2-path Vertex Cover Problem, is one of the best studied problems in graph theory. It is well-known to be $\mathcal{NP}$-hard. Brešar et al. reduced it to k-path Vertex Cover Problem (k-PVCP for short), which asks for a vertex set U

in a graph G of minimum cardinality such that $G - U$ contains no (not necessarily induced) path of length k. In this talk, we will show $\mathcal{NP}$-hardness of the k-PVCP in some graph classes based on this reduction and we will study the reduction of 3-PVCP to 2-PVCP. Using these results we obtain new bounds and a special graph class, where the 3-PVCP is solvable in polynomial time.

A Primal-Dual 3-Approximation Algorithm for Hitting 4-Vertex Paths

Eglantine Camby (joint work with Jean Cardinal, Mathieu Chapelle, Samuel Fiorini, and Gwenaël Joret)

We consider the problem of removing a minimum number of vertices of a given graph G so that the resulting graph does not contain the path P_k on k vertices as a subgraph. (Thus for $k = 2$ this is the vertex cover problem.) While for $k \in \{2, 3\}$ the problem admits a 2-approximation algorithm, nothing better than a trivial k-approximation is known for $k \geq 4$. Our main contribution is a 3-approximation algorithm in the case $k = 4$, that is, for hitting P_4's. The algorithm is inspired by the elegant primal-dual 2-approximation algorithm of Chudak, Goemans, Hochbaum, and Williamson (Operations Research Letters, 1998) for the feedback vertex set problem.

On the Möbius function of the quasi-consecutive pattern poset

Luca Ferrari (joint work with Antonio Bernini)

We define the quasi consecutive pattern poset by declaring $\sigma \leq \tau$ whenever the permutation τ contains an occurrence of the permutation σ in which all the entries are adjacent in τ except at most the first and the second. We then investigate the Möbius function of the quasi consecutive pattern poset and we completely determine it for those intervals $[\sigma, \tau]$ such that σ occurs precisely once in τ.

Restricted Steinhaus-Johnson-Trotter list
Ahmad Sabri (joint work with Vincent Vajnovszki)

We show that the restriction of Steinhaus-Johnson-Trotter Gray code, when restricted to some pattern avoiding permutations, still remains a (possibly less restricted) Gray code. By adapting SJT adjacent transposition based generating algorithm, we obtain an efficient generating algorithm for these Gray codes. These results complete the list of (pattern) restricted permutations that can be efficiently generated.

A homological characterization of planar graphs
Hein van der Holst

In this talk we give a homological characterization of planar graphs. A closure of a graph G is a certain cell complex $\mathcal{C}$ that can be associated with G. The deleted product of a closure $\mathcal{C}$ is the cell complex $\mathcal{C}^*$ obtained from $\mathcal{C} \times \mathcal{C}$ by removing all cells $\sigma \times \tau$ with σ and τ adjacent. The homological characterization is as follows: A graph is planar if and only if $H_3^{\mathrm{sym}}(\mathcal{C}^*)$ is trivial.

An update on sorting permutations by short block-moves
Luís Felipe I. Cunha (joint work with Luis Antonio B. Kowada, Rodrigo Hausen, and Celina M.H. de Figueiredo)

Sorting permutations by transpositions (SBT) is an important NP-hard problem in genome rearrangements. A restricted form of SBT, sorting permutations by short block-moves, is still open. In this paper we identify a family of permutations in which a shortest sequence of short block-moves is also a shortest sequence of transpositions that sorts those permutations, provide some sufficient conditions to determine the short block-moves distance, and also present an algorithm that sorts a given permutation by short block-moves, which implies a new upper bound for the short block-moves distance.

Rainbow connection and size of graphs
Ingo Schiermeyer

An edge-coloured connected graph G is called rainbow-connected if each pair of distinct vertices of G is connected by a path whose edges have distinct colours. The rainbow connection number of G, denoted by $rc(G)$, is the minimum number of colours such that G is rainbow-connected.

In this talk we will consider the following problem. *For all integers n and k compute and minimize the function $f(n,k)$ with the following property: If G has n vertices and at least $f(n,k)$ edges, then $rc(G) \leq k$.*

In this talk we will present several results for this problem.

Connected Tropical Subgraphs in Vertex-Colored Graphs
Jean-Alexandre Anglès d'Auriac (joint work with Nathann Cohen, Hakim El Maftouhi, Ararat Harutyunyan, Sylvain Legay, and Yannis Manoussakis)

A subgraph of a vertex-colored graph is said to be tropical whenever it contains each color of the original graph at least once. In this work we study the problem of finding a minimal connected tropical subgraph. We show that this problem is NP-Hard for trees, interval graphs and split graphs, but polynomial when the number of colors is logarithmic on the number of vertices of the graph. We give results that provide upper bounds for the order of the minimal connected tropical subgraph under various sufficient conditions, for example, minimal degree or number of edges. We finaly study sufficient and necessary conditions for a random graph to have a tropical subgraph such that each color is present precisely once.

Proper Hamiltonian Cycles in Edge-Colored Multigraphs
Leandro Montero (joint work with Raquel Águeda, Valentin Borozan, Raquel Díaz, and Yannis Manoussakis)

A c-edge-colored multigraph has each edge colored with one of the c available colors and no two parallel edges have the same color. A proper hamiltonian cycle is a cycle containing all the vertices of the multigraph such that no two adjacent edges have the same color. In this work we establish sufficient conditions for a multigraph to have a proper hamiltonian cycle, depending on several parameters such as the number of edges, the rainbow degree, etc.

Cycles avoiding a Color in Colorful Graphs
Dirk Meierling (joint work with Janina Müttel and Dieter Rautenbach)

The Ramsey numbers of cycles imply that every 2-edge-colored complete graph on n vertices contains monochromatic cycles of all lengths between 4 and at least $\frac{2}{3}n$. We generalize this result to $k \geq 3$ colors by showing that every k-edge-colored complete graph on $n \geq 6$ vertices contains $(k-1)$-edge-colored cycles of all lengths between 3 and at least $\left(\frac{2k-2}{2k-1} - \frac{2k-4}{\sqrt{n}}\right) n$.

Deciding Graph non-Hamiltonicity via a Closure Algorithm
Stephen G. Gismondi (joint work with Catherine E. Bell, Edward R. Swart and Nicholas R. Swart)

We present an heuristic algorithm that decides graph non-Hamiltonicity. All graphs are directed, each undirected edge regarded as a pair of counter directed arcs. Each of the $n!$ Hamilton cycles in a complete graph on $n+1$ vertices is mapped to an n-permutation matrix P where $p_{u,i} = 1$ if and only if the i^{th} arc in a cycle enters vertex u, starting and ending at vertex $n+1$. We first create exclusion set E by noting all arcs (u, v) not in G, sufficient to code precisely all cycles excluded from G i.e. cycles not in G use at least one arc not in G. Members are pairs of components of P, $\{p_{u,i}, p_{v,i+1}\}, i = 1, n-1$. A doubly stochastic-like relaxed LP formulation of the Hamilton cycle decision problem is constructed. Each $\{p_{u,i}, p_{v,i+1}\} \in E$ is coded as variable $q_{u,i,v,i+1} = 0$ i.e. shrinks the feasible region. We then implement the Weak Closure Algorithm (WCA) that tests necessary conditions of a matching, together with Boolean closure to decide 0/1 variable assignments. Each $\{p_{u,i}, p_{v,j}\} \notin E$ is tested for membership in E, and if possible, added to E ($q_{u,i,v,j} = 0$) to iteratively maximize $|E|$. If the WCA constructs E to be maximal, the set of all $\{p_{u,i}, p_{v,j}\}$, then G is decided non-Hamiltonian. Only non-Hamiltonian G share this maximal property. Ten non-Hamiltonian graphs (10 through 104 vertices) and 2000 randomized 31 vertex non-Hamiltonian graphs are tested and correctly decided non-Hamiltonian. For Hamiltonian G, the complement of E covers a matching, perhaps useful in searching for cycles. We also present an example where the WCA fails.

Hamiltonicity and Traceability of Locally Hamiltonian and Locally Traceable Graphs

Johan de Wet (joint work with Susan A. van Aardt)

We answer the following two questions which were posed by Pareek and Skupien in 1983:

1. Is 14 the smallest order of a connected locally hamiltonian nontraceable (LHNT) graph?

2. Is 9 the smallest order of a connected locally traceable nontraceable (LTNT) graph?

We develop a technique to construct planar and nonplanar connected locally hamiltonian nonhamiltonian (LHNH) graphs of every order greater than 10 and use this technique to derive some interesting properties of these graphs.

We also derive some analogous results for LTNT graphs.

Block Duplicate Graphs: Toughness and Hamiltonicity
Lilian Markenzon (joint work with Christina F. E. M. Waga)

The clique-based structure of chordal graphs allows the development of efficient solutions for many algorithmic problems. In this context, the minimal vertex separators play a decisive role. In this paper we present new results about block duplicate graphs, a subclass of chordal graphs, also called strictly chordal graphs. Based on a characterization which relies on properties of their minimal vertex separators, we present a linear time determination of the toughness of the class. This result yields a characterization of hamiltonian block duplicate graphs.

Hamiltonicity in squares of graphs revisited
Herbert Fleischner (joint work with Gek Ling Chia)

The first proof that the square of every 2-connected graph is hamiltonian, was achieved at the beginning of 1971 (published in 1974 only). Later on, Riha (1991) produced a much shorter proof, and an even shorter proof was found by Georgakopoulos (2009). However, it seems that the latter authors' methods do not extend to the range of graphs covered by the theory of EPS-graphs as developed by Fleischner, and Fleischner and Hobbs. An EPS-graph of G is a connected spanning subgraph of G which is the edge-disjoint union of an eulerian graph and a linear forest. As a matter of fact, in 1975 Fleischner and Hobbs determined the most general block-cutpoint structure a graph G may have such that $T(G)$ is hamiltonian; their proof depends on the existence of various EPS-graphs in a 2-connected graph.

In very recent joint work with Gek Ling Chia (University of Malaya) we determine the most general block-cutpoint structure a graph may have such that its square is hamiltonian connected. To achieve this we first showed that for any vertices x, y, u, v in a 2-connected graph G , there is a hamiltonian path in the square of G joining x and y and containing an edge of G incident with u and another such edge incident with v. In proving this result we have to deal with certain spanning subgraphs in 2-connected DT-graphs, which resemble EPS-graphs (in a DT-graph, every edge is incident to a vertex of degree two).

Path decompositions of triangle-free 5-regular graphs
Fábio Botler (joint work with Guilherme O. Mota and Yoshiko Wakabayashi)

A P_k-decomposition of a graph G is a set of edge-disjoint paths of G with k edges that cover the edge set of G. Kotzig (1957) proved that a 3-regular graph admits a P_3-decomposition if and only if it contains a perfect matching, and also asked what are the necessary and sufficient conditions for a $(2k+1)$-regular graph to admit a decomposition into paths with $2k+1$ edges. We partially answer this question for the case $k = 2$ by proving that the existence of a perfect matching is sufficient for a triangle-free 5-regular graph to admit a P_5-decomposition. This result contributes positively to the conjecture of Favaron, Genest, and Kouider (2010) that states that every 5-regular graph with a perfect matching admits a P_5-decomposition.

Decomposition of eulerian graphs into odd closed trails
Martin Škoviera (joint work with Edita Máčajová)

We show that an eulerian graph G admits a decomposition into k closed trails of odd length if and only if it contains at least k pairwise edge-disjoint odd circuits and $|E(G)| \equiv k \pmod 2$. We conjecture that a connected $2d$-regular graph of odd order with $d \geq 1$ admits a decomposition into d odd closed trails sharing a common vertex and verify the conjecture for $d \leq 3$. The case $d = 3$ is crucial for determining the flow number of a signed eulerian graph and the proof is surprisingly difficult.

On path-cycle decompositions of triangle-free graphs
Andrea Jiménez (joint work with Yoshiko Wakabayashi)

A path (resp. path-cycle) decomposition of a graph is a partition of its edge set into paths (resp. paths and cycles). About fifty years ago, according to Lovász, Gallai conjectured that every simple connected graph on n vertices admits a path decomposition of cardinality at most $\lceil n/2 \rceil$. Despite many attempts to prove Gallai's Conjecture, it remains unsolved. Lovász proved that connected graphs on n vertices admit a path-cycle decomposition of cardinality at most $\lfloor n/2 \rfloor$; this implies that Gallai's Conjecture is true for connected graphs in which all degrees are odd. In this work, we study conditions for the existence of $\geq$4-path-cycle decompositions; i.e., a path-cycle decomposition with elements of length at least 4. We characterize the class of triangle-free graphs with odd distance at least 3 that do not admit a $\geq$4-path-cycle decomposition. Then, with the help of a result of Harding et al., we transform $\geq$4-path-cycle decompositions into path decompositions with elements of average length at least 4. As a consequence, we prove that Gallai's Conjecture holds on a broad class of sparse graphs, which includes the class of triangle-free planar graphs with odd distance at least 3.

Decomposition of Complete Multigraphs into Stars and Cycles
Fairouz Beggas (joint work with Mohammed Haddad and Hamamache Kheddouci)

Let k be a positive integer. S_k and C_k denote respectively a star and a cycle of k edges. λK_n is the usual notation for the complete multigraph on n vertices and in which every edge is taken λ times. In this paper, we investigate necessary and sufficient conditions for the existence of the decomposition of λK_n into edges disjoint of stars S_k's and cycles C_k's.

Rainbow Colouring of Split Graphs
Deepak Rajendraprasad (joint work with L. Sunil Chandran and Marek Tesař)

A *rainbow path* in an edge coloured graph is a path in which no two edges are coloured the same. A *rainbow colouring* of a connected graph G is a colouring of the edges of G such that every pair of vertices in G is connected by at least one rainbow path. The minimum number of colours required to rainbow colour G is called its *rainbow connection number*. Between them, Chakraborty et al. and Ananth et al. have shown that for every integer k, $k \geq 2$, it is NP-complete to decide whether a given graph can be rainbow coloured using k colours.

A *split graph* is a graph whose vertex set can be partitioned into a clique and an independent set. Chandran and Rajendraprasad have shown that the problem of deciding whether a given split graph G can be rainbow coloured using 3 colours is NP-complete and further have described a linear time algorithm to rainbow colour any split graph using at most one colour more than the optimum. In this article, we settle the computational complexity of the problem on split graphs and thereby discover an interesting dichotomy. Specifically, we show that the problem of deciding whether a given split graph can be rainbow coloured using k colours is NP-complete for $k \in \{2, 3\}$, but can be solved in polynomial time for all other values of k.

Relaxed locally identifying coloring of graphs
Souad Slimani (joint work with Méziane Aïder and Sylvain Gravier)

A *locally identifying coloring* (*lid*-coloring) of a graph is a proper coloring such that the sets of colors appearing in the closed neighborhoods of any pair of adjacent vertices having distinct neighborhoods are distinct. Our goal is to study a *relaxed locally identifying coloring* (*rlid*-coloring) of a graph that is similar to locally identifying coloring for which the coloring is not necessary proper. We denote

by $\chi_{rlid}(G)$ the minimum number of colors used in a relaxed locally identifying coloring of a graph G.
In this paper, we prove that the problem of deciding that $\chi_{rlid}(G) = 3$ for a 2-degenerate planar graph G is NP-complete. We give several bounds of $\chi_{rlid}(G)$ and construct graphs for which these bounds are tightened. Studying some families of graphs allow us to compare this parameter with the minimum number of colors used in a locally identifying coloring of a graph G ($\chi_{lid}(G)$), the size of a minimum identifying code of G ($\gamma_{id}(G)$) and the chromatic number of G ($\chi(G)$).

Edge-Odd Graceful Labelings of (n, k)-kite, $F_{m,n}$ and the two Copies of a Graph
Sirirat Singhun

A graph G with q edges has an *edge-odd graceful labeling* if there is a bijection f from the edge set of G to the set $\{1, 3, 5, \ldots, 2q-1\}$ such that, when each vertex is assigned the sum of all the edges incident to it mod $2q$, the resulting vertex labels are distinct. A graph admitting an edge-odd graceful labeling is said to be *edge-odd graceful.* In this paper, we find families of edge-odd graceful graphs.

Vertex distinguishing colorings of graphs
Marius Woźniak

Let us consider a proper coloring f of edges in a simple graph $G = (V, E)$. Such a coloring defines for each vertex $x \in V$ the palette of colors, *i.e.*, the set of colors of edges incident with x, denoted by $S(x)$. Two vertices x and y are *similar* if $S(x) = S(y)$. The minimum number of colors required in a proper coloring f without two similar vertices is called the *vertex-distinguishing index*, and is denoted by vdi(G). The vertex-distinguishing index was introduced and studied (as "observability" of a graph) by Černý, Horňák and Soták and, independently, (as “strong coloring”) by Burris and Schelp.

In general, for some families of graphs, the vertex-distinguishing index can be much greater than the maximum degree. For instance, consider a vertex-distinguishing coloring of a cycle of length n with k colors. Since each palette is of size two, and the number of all possible palettes cannot be smaller than n, we have $\binom{k}{2} \geq n$. Hence, $\mathrm{vdi}(C_n) \geq \sqrt{2n}$.

However, if we distinguish the vertices in another way, namely by sets of color walks starting from vertices, not just by their palettes, then the number of colors we need is very close to the chromatic index.

On the proper orientation number of bipartite graphs
Phablo F. S. Moura (joint work with Julio Araujo, Nathann Cohen, Susanna F. de Rezende, and Frédéric Havet)

An *orientation* of a graph G is a digraph D obtained from G by replacing each edge by exactly one of the two possible arcs with the same endvertices. For each $v \in V(G)$, the *indegree* of v in D, denoted by $d_D^-(v)$, is the number of arcs with head v in D. An orientation D of G is *proper* if $d_D^-(u) \neq d_D^-(v)$, for all $uv \in E(G)$. The *proper orientation number* of a graph G, denoted by $\overrightarrow{\chi}(G)$, is the minimum of the maximum indegree over all its proper orientations. In this paper, we prove that $\overrightarrow{\chi}(G) \leq \left\lfloor \left(\Delta(G) + \sqrt{\Delta(G)} \right) /2 \right\rfloor + 1$ if G is a bipartite graph, and $\overrightarrow{\chi}(G) \leq 4$ if G is a tree.

It is well-known that $\overrightarrow{\chi}(G) \leq \Delta(G)$, for every graph G. However, we prove that deciding whether $\overrightarrow{\chi}(G) \leq \Delta(G) - 1$ is already an NP-complete problem. We also show that it is NP-complete to decide whether $\overrightarrow{\chi}(G) \leq 2$, for planar *subcubic* graphs G. Moreover, we prove that it is NP-complete to decide whether $\overrightarrow{\chi}(G) \leq 3$, for planar bipartite graphs G with maximum degree 5.

On the complexity of turning a graph into the analogue of a clique
Sergey Kirgizov (joint work with Julien Bensmail and Romaric Duvignau)

An orientation of an undirected graph G has weak diameter k if, for every pair $\{u, v\}$ of vertices of G, there is a directed path with length at most k joining u and v in either direction. We show that deciding whether an undirected graph admits an orientation with weak diameter k is NP-complete for every $k \geq 2$. This result implies the NP-completeness of deciding whether an undirected graph can be turned into the analogue of a clique for proper colouring of several augmented kinds of graphs.

Neighborhood Sequences of Graphs
Li-Da Tong

Let G be a graph, u be a vertex of G, and $B(u)$(or $B_G(u)$) be the set of u with all its neighbors in G. A sequence $(B_1, B_2, ..., B_n)$ of subsets of an n-set S is a *neighborhood sequence* if there exist a graph G with a vertex set S and a permutation $(v_1, v_2, ..., v_n)$ of S such that $B(v_i) = B_i$ for $i = 1, 2, ..., n$. The sequence $(v_1, v_2, ..., v_n)$ is called an *adjacent SDR* of $(B_1, B_2, ..., B_n)$. In this paper, we study the reconstruction problem from a neighborhood sequence and investigate the neighborhood sequences with two distinct adjacent SDRs.

Coloured degree sequences of graphs with at most one cycle
Anne Hillebrand (joint work with Colin McDiarmid and Alex Scott)

Coloured degree sequence problems, also known as edge-disjoint realisation and edge packing problems, have connections for example to discrete tomography, but are NP-hard to solve in general. Necessary and sufficient conditions are known for a demand matrix to be a coloured degree sequence of an edge coloured forest. We will give necessary and sufficient conditions for a demand matrix to be realisable by a graph with at most one cycle and discuss some related algorithmic questions.

Weighted Well-Covered Claw-Free Graphs
David Tankus (joint work with Vadim E. Levit)

A graph G is *well-covered* if all its maximal independent sets are of the same cardinality. Assume that a weight function w is defined on its vertices. Then G is w*-well-covered* if all maximal independent sets are of the same weight. For every graph G, the set of weight functions w such that G is w-well-covered is a *vector space*. Given an input $K_{1,3}$-free graph G, we present an $O\left(n^6\right)$ algortihm, whose input is a claw-free graph G, and output is the vector space of weight functions w, for which G is w-well-covered.

A graph G is *equimatchable* if all its maximal matchings are of the same cardinality. Assume that a weight function w is defined on the edges of G. Then G is w*-equimatchable* if all its maximal matchings are of the same weight. For every graph G, the set of weight functions w such that G is w-equimatchable is a vector space. We present an $O\left(m \cdot n^4 + n^5 \log n\right)$ algorithm, which receives an input graph G, and outputs the vector space of weight functions w such that G is w-equimatchable.

Augmenting Vertex for Maximum Independent Set in $S_{2,2,5}$-free Graphs
Ngoc C. Lê (joint work with Christoph Brause and Ingo Schiermeyer)

The method of augmenting vertex is a general approach to solve the maximum independent set (MIS for short) problem. This technique was used extensively for P_5-free graphs. Our objective is to employ this approach to develop polynomial time algorithms for the problem on $S_{2,2,5}$-free graphs, where $S_{i,j,k}$ is the graph consisting of three induced paths of lengths i, j, k with a common initial vertex.

Weighted Independent Sets in Classes of P_6-free Graphs
Frédéric Maffray (joint work with T. Karthick)

The Maximum Weight Independent Set (MWIS) problem on graphs with vertex weights asks for a set of pairwise nonadjacent vertices of maximum total weight. The complexity of the MWIS problem for P_6-free graphs and for $S_{1,2,2}$-free graphs is unknown. We give a proof for the solvability of the MWIS problem for (P_6, $S_{1,2,2}$, co-chair)-free graphs in polynomial time, by analyzing the structure of such graphs. These results extend some known results in the literature.

Asymptotic Surviving Rate of Trees with Multiple Fire Sources
Vitor Costa (joint work with Simone Dantas and Dieter Rautenbach)

For Hartnell's firefighter game with f vertices initially on fire and at most d defended vertices per round, the surviving rate $\rho(G, f, d)$ of a graph G is the average proportion of its vertices that can be saved in the game on G, where the average is taken over all equally likely sets of f fire sources. Cai et al. showed that $\rho(T, 1, 1) = 1 - O\left(\frac{\log n}{n}\right) = 1 - o(1)$ for every tree T of order n.

We study the maximum value $c(f, d)$ such that $\rho(T, f, d) \geq c(f, d) - o(1)$ for every tree T, that is, asymptotically, for the order tending to infinity, a proportion of $c(f, d)$ of the vertices of any tree can be saved on average. In this notation, Cai et al. result states $c(1, 1) = 1$. Our main results are that $c(f, 1) \geq 2\left(\frac{1}{3}\right)^{f+1}$ and that $\frac{4}{9} \leq c(2, 1) \leq \frac{3}{4}$.

The Optimal Rubbling Number of Ladders, Prisms and Möbius-ladders
Gyula Y. Katona (joint work with László F. Papp)

A pebbling move on a graph removes two pebbles at a vertex and adds one pebble at an adjacent vertex. Rubbling is a version of pebbling where an additional move is allowed. In this new move, one pebble each is removed at vertices v and w adjacent to a vertex u, and an extra pebble is added at vertex u. A vertex is reachable from a pebble distribution if it is possible to move a pebble to that vertex using rubbling moves. The optimal rubbling number is the smallest number m needed to guarantee a pebble distribution of m pebbles from which any vertex is reachable. We determine the optimal rubbling number of ladders ($P_n \Box P_2$), prisms ($C_n \Box P_2$) and Möblus-ladders.

Contraction Obstructions for Connected Graph Searching
Dimitris Zoros (joint work with Micah J. Best, Arvind Gupta, and Dimitrios M. Thilikos)

We consider the connected variant of the classic mixed search game where, in each search step, cleaned edges form a connected subgraph. We consider graph classes with bounded connected monotone mixed search number and we deal with the the question weather the obstruction set, with respect of the contraction partial ordering, for those classes is finite. In general, there is no guarantee that those sets are finite, as graphs are not well quasi ordered under the contraction partial ordering relation. In this paper we provide the obstruction set for $k = 2$. This set is finite, it consists of 174 graphs and completely characterizes the graphs with connected monotone mixed search number at most 2. Our proof reveals that the "sense of direction" of an optimal search searching is important for connected search which is in contrast to the unconnected original case.

A bound for the order of cages with a given girth pair
Julián Salas (joint work with Camino Balbuena)

The girth pair of a graph gives the length of a shortest odd and shortest even cycle. Harary and Kovács defined a generalization of $(k;g)$-cages to graphs with a girth pair (g,h). They proved that the order of a cage with a girth pair is bounded by two times the order of a cage with the large girth, i.e. $n(k;g,h) \leq 2n(k;h)$, they also proved that $n(k;h-1,h) \leq n(k;h)$, and conjectured that it always holds the inequality $n(k;g,h) \leq n(k;h)$. In this work we prove that their conjecture holds for odd girth $g < h$, and for even girth g and h sufficiently large. Assuming that that a bipartite $(k;g)$-cage exists when g is even.

Fault-tolerant bipancyclicity of Cayley graphs generated by transposition generating trees
Weihua He (joint work with Weihua Yang, Hengzhe Li, and Xiaofeng Guo)

A bipartite graph G is bipancyclic if G has a cycle of length l for every even l satisfied $4 \leq l \leq |V(G)|$. Let B be a transposition set and S_n be the symmetric group on $\{1,2,\ldots,n\}$. The Cayley graph $Cay(S_n,B)$ plays an important role for the study of Cayley graphs as interconnection networks. Let F be a subset of the edge set of $Cay(S_n,B)$, we show that $Cay(S_n,B) - F$ is bipancyclic if $Cay(S_n,B)$ is not a star graph, $n \geq 4$ and $|F| \leq n-3$.

Hamiltonian chordal graphs are not cycle extendible
Ben Seamone (joint work with Manuel Lafond)

A graph is cycle extendible if the vertices of any non-Hamiltonian cycle are contained in a cycle of length one greater. Hendry (1990) conjectured that every Hamiltonian chordal graph is cycle extendible. We construct an infinite family of counterexamples to this conjecture, and show that there exist counterexamples where the ratio of the length of a non-extendible cycle to the total number of vertices can be made arbitrarily small. With the conjecture settled, we turn our attention to determining what additional conditions are sufficient for Hendry's Conjecture to hold (e.g: connectivity, toughness, and forbidden induced subgraphs).

An edge variant of the Erdős-Pósa property
Jean-Florent Raymond (joint work with Ignasi Sau and Dimitrios M. Thilikos)

For every $r \in \mathbb{N}$, we denote by θ_r the multigraph with two vertices and r parallel edges. Given a graph G, we say that a subgraph H of G is *a model of* θ_r *in* G if H contains θ_r as a contraction. We prove that the following edge variant of the Erdős-Pósa property holds for every $r \geq 2$: if G is a graph and k is a positive integer, then either G contains a packing of k mutually edge-disjoint models of θ_r, or it contains a set X of $f_r(k)$ edges meeting all models of θ_r in G, for both $f_r(k) = O(k^2 r^3 \operatorname{polylog} kr)$ and $f_r(k) = O(k^4 r^2 \operatorname{polylog} kr)$.

Covering and packing pumpkin models
Dimitris Chatzidimitriou (joint work with Jean-Florent Raymond, Ignasi Sau, and Dimitrios M. Thilikos)

Let θ_r (the *r-pumpkin*) be the multi-graph containing two vertices and r parallel edges between them. We say that a graph is a *a* θ_r*-model* if it can be transformed into θ_r after a (possibly empty) sequence of contractions. We prove that there is a function $g : \mathbb{N} \to \mathbb{N}$ such that, for every two positive integers k and q, if G is a K_q-minor-free graph, then either G contains a set of k vertex-disjoint subgraphs (a θ_r*-model-vertex-packing*) each isomorphic to a θ_r-model or a set of $g(r) \cdot \log q \cdot k$ vertices (a θ_r*-model-vertex-cover*) meeting all subgraphs of G that are isomorphic to a θ_r-model. Our results imply a $O(\log OPT)$-approximation for the maximum (minimum) size of a θ_r-model packing (θ_r-model covering) of a graph G.

Proof of Berge's path partition conjecture for $k \geq \lambda - 3$

Dávid Herskovics

Let D be a digraph. A *path partition* of D is called k-optimal if the sum of k-norms of its paths is minimal. The *k-norm* of a path P is $\min(|V(P)|, k)$. Berge's path partition conjecture claims that for every k-optimal path partition $\mathcal{P}$ there are k disjoint stable sets orthogonal to $\mathcal{P}$. For general digraphs the conjecture has been proven for $k = 1, 2, \lambda - 1, \lambda$, where λ is the length of a longest path in the digraph. In this paper we prove the conjecture for $\lambda - 2$ and $\lambda - 3$.

On the number of palettes in edge-colorings of 4-regular graphs

Simona Bonvicini (joint work with Giuseppe Mazzuoccolo)

An edge-coloring of a simple graph G is an assignment of colors to the edges of G. An edge-coloring is proper if adjacent edges receive distinct colors. The set of colors assigned by a proper edge-coloring f to the edges incident to a vertex v of G is called the palette of v. The minimum number of palettes taken over all possible proper edge-colorings of G is called the palette index of G. We study the palette index of 4-regular graphs.

Extension from precoloured sets of edges

Ross J. Kang (joint work with Katherine Edwards, Jan van den Heuvel, and Jean-Sébastien Sereni)

We consider precolouring extension problems for proper edge-colouring, in an attempt to prove stronger versions of Shannon's and Vizing's theorems. We are most interested to extend a colouring from some arbitrarily precoloured matching. This turns out to be related to the notorious list colouring conjecture and other classic notions of choosability.

Fractional Colouring and Precolouring Extension of Graphs
Jan van den Heuvel (joint work with Daniel Král', Martin Kupec, Jean-Sébastien Sereni, and Jan Volec)

Suppose we are given a graph in which some vertices are already precoloured, and we want to extend this partial colouring to a colouring of the whole graph. Because of the precoloured vertices, we may need more colours than just the chromatic number. How many extra colours are needed under what conditions has been well-studied.

We consider the same problem in the context of fractional colourings. One way to define fractional colourings is as follows. We are given an interval $[0, k)$ of real numbers, and we need to assign to each vertex of subset of $[0, k)$ of measure one so that adjacent vertices receive disjoint subsets. The fractional chromatic number is the minimum k for which this is possible.

Again assume that certain vertices are already precoloured (i.e., are already assigned a subset of measure one). If we further assume some knowledge about the precoloured vertices (say they are far apart), what value of k is required to guarantee that we can always extend this partial colouring to a fractional colouring of the whole graph? The answer to this questions shows a surprising dependence on the fractional chromatic number of the graph under consideration.

On bipartization of cubic graphs by removal of an independent set
Hanna Furmańczyk (joint work with Marek Kubale)

We study a new problem for cubic graphs: bipartization of a cubic graph Q by deleting sufficiently large independent set I. It can be expressed as follows: *Given a connected n-vertex cubic graph $Q = (V, E)$ with independence number $\alpha(Q)$, does Q contain an independent set I of size k such that $Q - I$ is bipartite?* We prove constructively that for $\alpha(Q) \geq 4n/10$ the answer is affirmative for $\lfloor n/3 \rfloor \leq k \leq \alpha(G)$. It remains an open question if a similar construction is possible for $\alpha(Q) < 4n/10$.

We show that this problem with $\alpha(Q) \geq 4n/10$ can be related to semi-equitable graph 3-coloring, where one color class is of size k, $\lfloor n/3 \rfloor \leq k \leq \alpha(G)$, and the subgraph induced by the remaining vertices is equitably 2-colored. This means that Q has a coloring of type $(k, \lfloor (n-k)/2 \rfloor, \lceil (n-k)/2 \rceil)$.

On the k-independence number in graphs
Mostafa Blidia (joint work with Ahmed Bouchou)

For an integer $k \geq 1$ and a graph $G = (V, E)$, a subset S of V is k-independent if every vertex in S has at most $k-1$ neighbors in S. The k-independent number $\beta_k(G)$ is the maximum cardinality of a k-independent set of G. In this work, we study relations between $\beta_k(G)$, $\beta_j(G)$ and the domination number $\gamma(G)$ in a graph G where $1 \leq j < k$. Also we give some characterizations of extremal graphs.

Reconfiguring Independent Sets in Cographs
Marthe Bonamy (joint work with Nicolas Bousquet)

Let k be an integer. Two stable sets of a graph are *adjacent* if they differ on exactly one vertex (*i.e.* we can tranform one into the other by adding or deleting a vertex). We consider the reconfiguration graph $TAR_k(G)$ on the set of stable sets of size at least k in a graph G, with the above notion of adjacency. Here we provide a polynomial-time algorithm to decide whether $TAR_k(G)$ is connected when G is a cograph, thus solving an open question of Bonsma 2014.

A New Game Invariant of Graph: the Game Distinguishing Number
Simon Schmidt (joint work with Sylvain Gravier, Kahina Meslem, and Souad Slimani)

The distinguishing number of a graph G is a symmetry related graph invariant whose study started a decade ago. The distinguishing number $D(G)$ is the least integer d such that G has a

d-distinguishing coloring. A d-distinguishing coloring is a coloring $c : V(G) \to \{1, ..., d\}$ invariant only under the trivial automorphism. In this paper, we introduce a game variant of this invariant. The distinguishing game is a game with two players, the Gentle and the Rascal, with antagonist goals. This games is played on a graph G with a set of $d \in \mathbb{N}^*$ colors. Alternatively, the two players choose a vertex of G and color it with one of the d colors. The game ends when all the vertices have been colored. Then the Gentle wins if the coloration is d-distinguishing and the Rascal wins otherwise. This game leads to a definition of two new invariants for a graph G. Those invariants are the minimum numbers of colors needed to ensure that the Gentle has a winning strategy. We will compute those numbers for several classes of graphs, in particular, cycles, hypercubes and some cartesian products of complete graphs. We also defined a class of graphs, the involutive graphs, for which the game distinguishing number is at most quadratic in the classical distinguishing invariant.

Upper bounds on the game domination number
Csilla Bujtás

In the domination game, two players called Dominator and Staller alternately choose a vertex of a graph G and take it into a set D. The number of vertices dominated by the set D must increase in each single turn and the game ends when D becomes a dominating set of G. Dominator aims to minimize, whilst Staller aims to maximize the number of turns (or equivalently, the size of the dominating set D obtained at the end). Assuming that Dominator starts and both players play optimally, the number of turns is called the game domination number $\gamma_g(G)$ of G.

Kinnersley, West and Zamani verified that $\gamma_g(G) \leq 7n/11$ holds for every isolate-free n-vertex forest G and they conjectured that the sharp upper bound is only $3n/5$. Here we prove the 3/5-conjecture for forests in which no two leaves are at distance 4 apart, moreover we establish a new upper bound $\gamma_g(G) \leq 5n/8$, which is valid for every isolate-free forest G. Our proof technique is based on a value-assignment to the vertices and on a greedy-like strategy prescribed for Dominator. This approach is also applied for graphs in general (without structural restrictions), and we obtain a significant improvement on the previously known general upper bound $\lceil 7n/10 \rceil$.

Extremal properties of flood-filling games
Dominik K. Vu (joint work with Kitty Meeks)

We consider extremal questions related to the combinatorial game Free-Flood-It, in which players aim to make a coloured graph monochromatic with the minimum possible number of flooding operations; our goal is to determine, for specified graphs, the maximum number of moves that may be required when taken over all possible colourings. We give two general upper bounds on this quantity, which we show to be tight for particular classes of graphs, and determine this maximum number of moves exactly when the underlying graph is a path, cycle, or a blow-up of a path or cycle.

Minimum size extensible graphs for (near) perfect matchings
Christophe Picouleau (joint work with Marie-Christine Costa and Dominique de Werra)

We define as *extensible* a graph G such that for every pair u, v of non adjacent vertices it is possible to extend the non-edge uv to a perfect (or near perfect) matching using only edges of G that are not incident to u or v. For every order n of G we give $Ext(n)$ the minimum size of an extensible graph.

Equimatchable factor-critical graphs and graphs with independence number 2

Michal Kotrbčík (joint work with Eduard Eiben

A graph is equimatchable if any its matching is a subset of a maximum matching. It is well known that any 2-connected equimatchable graph is either bipartite, or factor critical, and that these two classes are disjoint. This paper provides a description of k-connected factor-critical equimatchable graphs with respect to their k-cuts. As our main result we prove that if a k-connected factor-critical graph has at least $2k + 3$ vertices and a k-cut S such that $G - S$ has two components with size at least 3, then $G - S$ has exactly two components and both are complete graphs. Furthermore, we show that if $k \geq 4$, then all such graphs have independence number 2.

Connected f-Factors of *Large* Minimum Degree in Polynomial Time

C. S. Rahul (joint work with N. S. Narayanaswamy)

We present results on the connected f-factor problem obtained as part of an exploration to identify a dichotomy result based on the nature of f. It is well known that when $f(v)$ is a constant for all $v \in V(G)$, the problem is NP-complete. Using the Tutte's f-factor algorithm, it is straightforward to see that checking for a connected f-factor can be done in polynomial time when $f(v) \geq \lceil \frac{n}{2} \rceil - 1, v \in V(G)$. We show that given a graph G containing n vertices, the problem of deciding whether G has a connected f-factor is polynomial time solvable if $f(v) \geq \lceil \frac{n}{2.5} \rceil, v \in V(G)$. The algorithm is obtained by showing that if Tutte's f-factor algorithm gives a factor with 2 components, and if there is a connected f-factor, then there is one of diameter 3.

Fast recognition of chair-free graphs

Mihai Talmaciu (joint work with Victor Lepin)

We give a characterization of *Chair*-free graphs and *Bull*-free using weak decomposition. We also give recognition algorithms for *chair*, *bull*-free graphs, comparable to the available ones as execution and we determine the combinatorial optimization numbers in efficient time. We construct a biclique partition.

Isolating highly connected induced subgraphs

Nicolas Trotignon (joint work with Irena Penev and Stéphan Thomassé)

We prove that any graph G with minimum degree greater than $2k^2 - 1$ has a $(k+1)$-connected induced subgraph H such that the number of vertices of H that have neighbors outside of H is at most $2k^2 - 1$. This generalizes a classical result of Mader stating that a high minimum degree implies a highly connected subgraph. We give several variants of our result, and for each of them, we give asymptotics for the bounds.

It was proven by Alon, Kleitman, Saks, Seymour and Thomassen that in a graph of high chromatic number, there exists an induced subgraph with high connectivity and high chromatic number. Our results give a new proof of this theorem with a better bound.

On (claw, even hole)-free graphs

Chính T. Hoàng (joint work with Kathie Cameron and Steven Chaplick)

An even hole is an induced even cycle. Even-hole-free graphs generalize chordal graphs. We prove that claw-free even-hole-free graphs can be decomposed by clique-cutsets into, essentially, proper circular-arc graphs. This provides the basis for our algorithms for recognizing and colouring these graphs. Our recognition algorithm is more efficient than known algorithms for recognizing even-hole-free graphs. Minimum colouring of claw-free graphs is NP-hard and the complexity of colouring even-hole-free graphs is unknown, but our algorithm colours claw-free even-hole-free graphs in $O(n^3)$ time.

Solution of Vizing's Problem on Interchanges for Graphs with Maximum Degree 4 and Related Results

Armen S. Asratian (joint work with Carl Johan Casselgren)

Let G be a Class 1 graph with maximum degree 4 and let $t \geq 5$ be an integer. We show that any proper t-edge coloring of G can be transformed to any proper 4-edge coloring of G using only transformations on 2-colored subgraphs (so-called interchanges). This settles the smallest previously unsolved case of a well-known problem of Vizing on interchanges, posed in 1965. Using our result we give an affirmative answer to a question of Mohar for two classes of graphs: we show that all proper 5-edge colorings of a Class 1 graph with maximum degree 4 are Kempe equivalent, that is, can be transformed to each other by interchanges, and that all proper 7-edge colorings of a Class 2 graph with maximum degree 5 are Kempe equivalent.

Contraction Blockers

Bernard Ries (joint work with Öznur Yaşar Diner, Daniël Paulusma and Christophe Picouleau)

We consider the following problem: can a certain graph parameter of some given graph be reduced by at most d for some integer d via at most k edge contractions for some given integer k? We consider three graph parameters: the chromatic number, clique number and independence number. For each of these graph parameters we show that, when d is part of the input, this problem is polynomial-time solvable on P_4-free graphs and NP-complete for split graphs. As split graphs form a subclass of P_5-free graphs, both results together give a complete complexity classification for P_ℓ-free graphs.

4-Critical Graphs of Girth ≥ 5 have at least $(\frac{5}{3}+\epsilon)|V(G)|$ edges
Luke Postle

Dirac introduced the notion of a k-critical graph, a graph that is not $(k-1)$-colorable but every proper subgraph of which is. Motivated by Hajos' construction, Ore conjectured a bound on the minimum density of a k-critical graph. Kostochka and Yancey recently proved Ore's conjecture for almost all graphs. Their proof for $k=4$ is especially short and implies Grotzch's theorem that every triangle-free planar graph is 3-colorable.

In this paper, we prove a strengthening of their result by showing that the minimum density can be increased if we subtract a factor proportional to the maximum number of vertex disjoint cycles of length at most four. As a corollary, we find that the density of 4-critical graphs of girth at least five is at least $\frac{5}{3}+\epsilon$ for some constant $\epsilon > 0$. This implies a theorem of Thomassen which states that for every surface, the number of 4-critical graphs of girth at least five embeddable on that surface is finite. Indeed, this even implies the stronger result of Dvorak, Kral and Thomas that the number of vertices of a 4-critical of girth at least five is at most linear in its genus. Furthermore, it provides a short proof of the fact that a planar graph whose cycles of length at most four are far enough apart is 3-colorable, which is also a result of Dvorak, Kral and Thomas.

On an anti-Ramsey threshold for sparse graphs with one triangle
Guilherme O. Mota (joint work with Yoshiharu Kohayakawa and Pavlos B. Konstadinidis)

For graphs G and H, let $G \xrightarrow[\mathrm{p}]{\mathrm{rb}} H$ denote the property that for every *proper* edge-colouring of G (with an arbitrary number of colours) there is a *totally multicoloured*, or *rainbow*, copy of H in G, that is, a copy of H with no two edges of the same colour. It was proved that for every graph H, the threshold function $p_H^{\mathrm{rb}} = p_H^{\mathrm{rb}}(n)$

of this property for the binomial random graph $G(n,p)$ is asymptotically at most $n^{-1/m^{(2)}(H)}$. Here we prove that there exists a fairly rich, infinite family of graphs F containing a triangle such that if $p \geq Dn^{-\beta}$ for suitable constants $D = D(F) > 0$ and $\beta = \beta(F)$, where $\beta > 1/m^{(2)}(F)$, then $G(n,p) \xrightarrow[\mathrm{p}]{\mathrm{rb}} F$ almost surely. In particular, $p_F^{\mathrm{rb}} \ll n^{-1/m^{(2)}(F)}$ for any such graph F.

The density Turán problem for some unicyclic graphs
Halina Bielak (joint work with Kamil Powroźnik)

Let H be a graph with n vertices and let $G[H]$ denotes a blow-up graph of the graph H defined in the following way. Replace each vertex i of H by a cluster A_i and connect vertices between the clusters A_i and A_j (not necessarily all) if vertices i and j are adjacent in H, $i,j \in V(H)$. Let define the edge density between A_i and A_j by the formula $d(A_i,A_j) = \frac{e(A_i,A_j)}{|A_i||A_j|}$, where $e(A_i,A_j)$ denotes the number of edges between the clusters A_i and A_j. The graph H is *a transversal* of $G[H]$ if H is a subgraph of G[H] such that we have a homomorphism $\phi : V(H) \to V(G[H])$ for which $\phi(i) \in A_i$ for all $i \in V(H)$.

We are interested in the following problem. For each edge $e = \{i,j\} \in E(H)$ a density γ_e is given. Instead of $e = \{i,j\}$ we shortly write $e = ij$ and $\gamma_e = \gamma_{ij}$. We have to decide whether the set of densities $\{\gamma_e\}_{e \in E(H)}$ ensure the existence of the graph H as a transversal or we can construct a blow-up graph G[H] such that $d(A_i,A_j) \geq \gamma_{ij}$, but it does not induce the graph H as a transversal. We study the problem for some family of unicyclic graphs H. We give an efficient algorithm to decide whether a given set of edge densities ensures the existence of a factor H in a blow-up graph $G[H]$ or does not ensure, where H is a unicyclic graph with the cycle C_3. We extend the results of Csikvari and Nagy presented in *The density Turán Problem* (Combinatorics, Probability and Computing 21 (2012), 531-553).

Universal Spacings for the 3-Dimensional VLSI Routing in the Cube
Attila Kiss (joint work with András Recski)

In previous works some polynomial time algorithms were presented for special cases of the 3-Dimensional VLSI Routing problem. Solutions were given to problems when all the terminals are either on a single face (SALP - Single Active Layer Problem) or on two opposite faces (3DCRP - 3-Dimensional Channel Routing Problem) or on two adjacent faces (3DΓRP - 3-Dimensional Gamma Routing Problem) of a rectangular cuboid. We prove that combining these algorithms one can solve any given problem on cubes and we give some polynomial time algorithms to find these solutions.

The Pseudograph (r, s, a, t) - threshold number
Anitha Rajkumar (joint work with Anthony J.W. Hilton)

For $d \geq 1, s \geq 0$, a $(d, d+s)$ - graph is a graph whose degrees all lie in the interval $\{d, d+1, \ldots, d+s\}$. For $r \geq 1, a \geq 0$, an $(r, r+a)$ - factor of a graph G is a spanning $(r, r+a)$ - subgraph of G. An $(r, r+a)$ - factorization of a graph G is a decomposition of G into edge-disjoint $(r, r+a)$ - factors. A *pseudograph* is a graph which may have multiple edges and may have multiple loops. A loop counts two towards the degree of the vertex it is on. A *multigraph* here has no loops.

For $t \geq 1$, let $\pi(r, s, a, t)$ be the least integer such that, if $d \geq \pi(r, s, a, t)$ then every $(d, d+s)$ - pseudograph G has an $(r, r+a)$ - factorization into x $(r, r+a)$ - factors for at least t different values of x. We call $\pi(r, s, a, t)$ the pseudograph (r, s, a, t) - threshold number. Let $\mu(r, s, a, t)$ be the analogous integer for multigraphs. We call $\mu(r, s, a, t)$ the multigraph (r, s, a, t) - threshold number. A *simple graph* has at most one edge between any two vertices and has no loops. We let $\sigma(r, s, a, t)$ be the analogous integer for simple graphs. We call $\sigma(r, s, a, t)$ the simple graph (r, s, a, t) - threshold number.

In this paper we give the precise value of the pseudograph $\pi(r, s, a, t)$ - threshold number for each value of r, s, a and t. We also use this to give good bounds for the analogous simple graph and multigraph threshold numbers $\sigma(r, s, a, t)$ and $\mu(r, s, a, t)$.

A proof of the Tuza-Vestergaard Conjecture
Christian Löwenstein (joint work with Anders Yeo)

The transversal number, denoted $\tau(H)$, of a hypergraph H is the minimum number of vertices that intersect every edge. A hypergraph is k-uniform if every edge has size k. Tuza and Vestergaard conjectured in 2002 that every 3-regular 6-uniform hypergraph of order n has a transversal of size at most $n/4$. We will prove this conjecture.

Polynomial-time perfect matchings in dense hypergraphs
Fiachra Knox (joint work with Peter Keevash and Richard Mycroft)

Let H be a k-graph on n vertices, with minimum codegree at least $n/k + cn$ for some fixed $c > 0$. We describe a polynomial-time algorithm which finds either a perfect matching in H or a certificate that none exists. This essentially solves a problem of Karpiński, Ruciński and Szymańska; Szymańska previously showed that this problem is NP-hard for a minimum codegree of $n/k - cn$. Our algorithm relies on a theoretical result of independent interest, in which we characterise any such hypergraph with no perfect matching using a family of lattice-based constructions.

Limitations of the theory of direct type algorithms
Aleksandr Maksimenko

We discuss the complexity of linear search algorithms (LSAs) for solving combinatorial optimization problems. More precisely, we consider the class of direct type algorithms that is a restricted version of LSAs. The restriction is defined in terms of a polytope $P(X)$ associated with the appropriate combinatorial optimization problem X. In particular, the clique number $\omega(X)$ of the 1-skeleton of a polytope $P(X)$ is the lower bound for the complexity of X in the class of direct type algorithms. We show that the Hungarian algorithm for the assignment problem is not a direct type algorithm. Hence, this class of algorithms is not large. On the other hand, we prove that any problem X can be transformed to a more complex problem Y with the clique number $\omega(X) = 2$.

Structures with no finite monomorphic decomposition. Application to the profile of hereditary classes
Djamila Oudrar (joint work with Maurice Pouzet)

We present a structural approach of some results about jumps in the behavior of the profile (alias generating functions) of hereditary classes of finite structures. We start with the following notion due to N. Thiéry and the second author. A *monomorphic decomposition* of a relational structure R is a partition of its domain $V(R)$ into a family of sets $(V_x)_{x\in X}$ such that the restrictions of R to two finite subsets A and A' of $V(R)$ are isomorphic provided that the traces $A\cap V_x$ and $A'\cap V_x$ have the same size for each $x \in X$. Let $\mathscr{S}_\mu$ be the class of relational structures of signature μ which do not have a finite monomorphic decomposition. We show that if a hereditary subclass $\mathscr{D}$ of $\mathscr{S}_\mu$ is made of binary structures or of ordered relational structures then it contains a finite subset $\mathfrak{A}$ such that every member of $\mathscr{D}$ embeds some member of $\mathfrak{A}$. From the description of the members R of $\mathfrak{A}$ we show that if R is ordered then the profile of the age $\mathcal{A}(R)$ of R (made of finite substructures of R), is at least exponential. We deduce that a hereditary class of finite ordered structures whose profile is not bounded by a polynomial is at least exponential. This result is a part of a much deeper classification obtained by Balogh, Bollobás and Morris for ordered graphs and ordered hypergraphs.

Geometric Extensions of Cutwidth in any Dimension
Spyridon Maniatis (joint work with Menelaos I. Karavelas, Dimitrios M. Thilikos and Dimitris Zoros)

We define a multi-dimensional geometric extension of cutwidth. A graph has d-cutwidth at most k if it can be embedded in the d-dimensional euclidean space so that no hyperplane can intersect more than k of its edges. We prove a series of combinatorial results on d-cutwidth which imply that for every d and k, there is a linear time algorithm checking whether the d-cutwidth of a graph G is at most k.

SEFE = C-Planarity?
Giordano Da Lozzo (joint work with Patrizio Angelini)

In this paper we deepen the understanding of the connection between two long-standing Graph Drawing open problems, that is, Simultaneous Embedding with Fixed Edges (SEFE) and Clustered Planarity (C-Planarity). In his GD'12 paper Marcus Schaefer presented a reduction from C-Planarity to SEFE of two planar graphs (SEFE-2). We prove that a reduction exists also in the opposite direction, if we consider instances of SEFE-2 in which the intersection graph is connected. We pose as an open question whether the two problems are polynomial-time equivalent.

Separation dimension of sparse graphs
Manu Basavaraju (joint work with L. Sunil Chandran, Rogers Mathew, and Deepak Rajendraprasad)

The *separation dimension* of a graph G is the smallest natural number k for which the vertices of G can be embedded in $\mathbb{R}^k$ such that any pair of disjoint edges in G can be separated by a hyperplane normal to one of the axes. Equivalently, it is the smallest possible cardinality of a family $\mathcal{F}$ of permutations of the vertices of G such that for any two disjoint edges of G, there exists at least one permutation in $\mathcal{F}$ in which all the vertices in one edge precede those in the other. In general, the maximum separation dimension of a graph on n vertices is $\Theta(\log n)$. In this article, we focus on sparse graphs and show that the maximum separation dimension of a k-degenerate graph on n vertices is $O(k \log\log n)$ and that there exists a family of 2-degenerate graphs with separation dimension $\Omega(\log\log n)$. We also show that the separation dimension of the graph $G^{1/2}$ obtained by subdividing once every edge of another graph G is at most $(1 + o(1)) \log\log \chi(G)$ where $\chi(G)$ is the chromatic number of the original graph.

On the Inverse of the Adjacency Matrix of a Graph
Irene Sciriha (joint work with Alexander Farrugia and John Baptist Gauci)

A real symmetric matrix $\mathbf{G}$ with zero diagonal encodes the adjacencies of the vertices of a graph G with weighted edges and no loops. A graph associated with a $n \times n$ non–singular matrix with zero entries on the diagonal such that all its $(n-1) \times (n-1)$ principal submatrices are singular is said to be a NSSD. We show that the class of NSSDs is closed under taking the inverse of $\mathbf{G}$. We present results on the nullities of one– and two–vertex deleted subgraphs of a NSSD. It is shown that a necessary and sufficient condition for two–vertex deleted subgraphs of G and of the graph $\Gamma(\mathbf{G}^{-1})$ associated with $\mathbf{G}^{-1}$ to remain NSSDs is that the submatrices belonging to them, derived from $\mathbf{G}$ and $\mathbf{G}^{-1}$, are inverses. Moreover, an algorithm yielding what we term plain NSSDs is presented. This algorithm can be used to determine if a graph G with a terminal vertex is *not* a NSSD.

The Adjacency Matrices of Complete and Nutful Graphs
Alexander Farrugia

A real symmetric matrix $\mathbf{G}$ with zero entries on its diagonal is an adjacency matrix associated with a graph G (with weighted edges and no loops) if and only if the non–zero entries correspond to edges of G. An adjacency matrix $\mathbf{G}$ belongs to a generalized–nut graph G if every entry in a vector in the nullspace of $\mathbf{G}$ is non–zero. A graph G is termed NSSD if it corresponds to a non–singular adjacency matrix $\mathbf{G}$ with a singular deck $\{\mathbf{G} - \mathbf{v}\}$, where $\mathbf{G} - \mathbf{v}$ is the submatrix obtained from $\mathbf{G}$ by deleting the v^{th} row and column. An NSSD G whose deck consists of generalized–nut graphs with respect to $\mathbf{G}$ is referred to as a $\mathbf{G}$–nutful graph. We prove that a $\mathbf{G}$–nutful NSSD is equivalent to having a NSSD with $\mathbf{G}^{-1}$ as the adjacency matrix of the complete graph. If the entries of $\mathbf{G}$ for a $\mathbf{G}$–nutful graph are restricted to 0 or 1, then the graph is known as nuciferous, a concept that has arisen in the context of the quantum mechanical theory of the conductivity of non–singular Carbon molecules according to the SSP model. We characterize nuciferous graphs by their inverse and the nullities of their one– and two–vertex deleted subgraphs. We show that a $\mathbf{G}$–nutful graph is a NSSD which is either K_2 or has no pendant edges. Moreover, we reconstruct a labelled NSSD either from the nullspace generators of the ordered one–vertex deleted subgraphs or from the determinants of the ordered two–vertex deleted subgraphs.

Edge–weighted Complete Graphs With Zero Diagonal Inverse

John Baptist Gauci

If the inverse of a non–singular real symmetric matrix that represents an edge–weighted graph with no loops has zero diagonal, then the inverse itself is the matrix of a loopless weighted graph. The matrices associated with the one–vertex deleted subgraphs of such graphs are singular. This family of *Non–Singular graphs with a Singular Deck* (NSSD) has the remarkable property that it is closed under taking the inverse. We show that such graphs always exist if their number of vertices is at least six. In particular, we describe a construction of a substructure of complete graphs (termed *complete NSSD*) that have this property, and whose inverse turn out to be other complete graphs with (possibly different) edge–weights. This construction is realised via the spectral properties of circulant matrices.

Recognition of dynamic circle graphs

Christophe Paul (joint work with Christophe Crespelle and Emeric Gioan)

A *circle graph* is the intersection graph of a set of chords in a circle, this geometric representation of the graph being called a *chord diagram*. A given circle graph may be represented by many different chord diagrams but it is well-known that the set of all possible diagrams can be represented in $O(n)$ space by using the split decomposition. In this paper, we propose an $O(n)$-time algorithm that, given a n-vertex circle graph, maintains the split-decomposition-based representation of a circle graph under vertex deletion and vertex insertion (or asserts that the resulting graph is not circle).

Paths in a Tree: Structural Properties

Pierre Duchet

A unified framework is given to study several classes of hypergraphs arising from families of paths in a (finite) tree. A path can be viewed as a series of vertices or as a collection of edges. Since typically 3 kinds of trees may be involved (undirected trees, directed trees, and rooted trees) this leads to 6 main classes of "tree path hypergraphs", we name by the generic term of *lineations*. The intersection graphs of lineations, are understood quite well, with one notable exception : the intersection graphs of undirected edge lineations are NP-complete to recognize. On the other hand, the hypergraphic structure of these classes remained widely unexplored. The present paper gives a structural characterization for each class.

In each case, a list of excluded configurations and a fast algorithm can be derived (even in case of undirected edge-lineations)

Vertex-lineations (results obtained in collaboration with Z. Arami) are viewed as special classes of *arboreal* hypergraphs (= families of vertex-subtrees in a tree). Keysteps are the use of a parsimonious description of all trees that realize a given arboreal hypergraph, and the statement of some necessary balance condition.

For edge-lineations, the results follow from a separation criterion obtained by localization of the balance condition. A generalization to k-paths in k-trees can be given: associating to each vertex of a generic hypergraph $\mathscr{H}$ an appropriate "*separation-graph*" $S(\mathscr{H})$ and we prove that $\mathscr{H}$ is realizable as a collection of k-paths in some k-tree iff $S(\mathscr{H})$ is k-colorable. As a byproduct (the case $k = 2$) we obtain a new characterization of graphic matroids, with an $\mathrm{O}(n^4)$recognition algorithm.

The combinatorics of web worlds and web diagrams
Mark Dukes

We introduce and study a new combinatorial object called a *web world*. A web world consists of a set of diagrams that we call *web diagrams*. The motivation for introducing these comes from particle physics, where web diagrams arise as particular types of Feynman diagrams describing scattering amplitudes in non-Abelian gauge (Yang-Mills) theories. The *web world* of a web diagram is the set of all web diagrams that result from permuting the order in which endpoints of edges appear on a peg. To each web world we associate two matrices called the *web-colouring matrix* and *web-mixing matrix*, respectively. The entries of these matrices are indexed by ordered pairs of web diagrams (D_1, D_2), and are computed from those colourings of the edges of D_1 that yield D_2 under a certain transformation determined by each colouring.

We show that colourings of a web diagram (whose constituent indecomposable diagrams are all unique) that lead to a reconstruction of the diagram are equivalent to order-preserving mappings of certain partially ordered sets (posets) that may be constructed from the web diagrams. For web worlds whose web graphs have all edge labels equal to 1, the diagonal entries of web-mixing and web-colouring matrices are obtained by summing certain polynomials determined by the descents in permutations in the Jordan-Hölder set of all linear extensions of the associated poset. We give two tri-variate generating functions, recording statistics that keep track of the number of pegs, the number of edges and the number of pairs of pegs that are joined by some edge. We also obtain an expression for the number of different web diagrams in a given web world, in terms of entries of a matrix that represents the web world. Three special web worlds are examined in great detail, and the traces of the web-mixing matrices calculated in each case.

Equivalence classes of Dyck paths modulo some statistics
Armen Petrossian (joint work with Jean-Luc Baril)

We define new equivalence relations on the set $\mathcal{D}_n$ of Dyck paths relatively to the three statistics of double rises, peaks and valleys. Two Dyck paths are r-equivalent (respectively p-equivalent and v-equivalent) whenever the positions of their double rises (respectively peaks and valleys) are the same. Then, we provide generating functions for the numbers of r-, p- and v-equivalence classes.

Counting Unlabelled Planar Graphs and Conjectures from String Theory
Shiroman Prakash (joint work with V. Gurucharan and Harsh Khandelwal)

The problem of counting unlabelled planar graphs has proven very challenging.

By counting unlabelled planar graphs, we mean obtaining an asymptotic estimate for the number of unlabelled planar graphs with n vertices with n large, as well as asking other statistical questions about unlabelled planar graphs with a large number of vertices. While the problem of counting labelled planar graphs has been solved fairly recently, the unlabelled case remains an outstanding unsolved combinatorial problem. One reason the unlabelled case is more difficult is that a generic unlabelled planar graph may have a fairly large automorphism group.

Planar graphs arise in a variety of contexts; perhaps most famously (for physicists), in the study of quantum field theories based on gauge groups with rank N, in the large-N limit, as was discovered by 't Hooft many years ago.

In this talk, we will describe some attempts at connecting combinatorial problems related to counting unlabelled planar graphs with very-well established conjectures about the behaviour of large-N gauge theories at strong coupling arising from string theory.

Minor relations for quadrangulations on the projective plane
Shin-ichi Yonekura (joint work with Naoki Matsumoto and Atsuhiro Nakamoto)

A *quadrangulation* on a surface is a map of a simple graph on the surface with each face quadrilateral. In this paper, we prove that for any bipartite quadrangulation G on the projective plane, there exists a sequence of bipartite quadrangulations on the same surface $G = G_1, G_2, \ldots, G_n$ such that

(i) G_{i+1} is a minor of G_i with $|G_i| - 2 \leq |G_{i+1}| \leq |G_i| - 1$, for $i = 1, \ldots, n-1$,

(ii) G_n is isomorphic to either $K_{3,4}$ or $K_{4,4}^{--}$,

where $K_{4,4}^{--}$ is the graph obtained from $K_{4,4}$ by removing two independent edges. In order to prove the theorem, we use two local reductions for quadrangulations which transform a quadrangulation Q into another quadrangulation Q' with $Q \geq_m Q'$ and

$1 \leq |Q| - |Q'| \leq 2$. Moreover, we prove a similar result for non-bipartite quadrangulations on the projective plane.

Book-embeddings of graphs on the projective plane
Kenta Ozeki (joint work with Atsuhiro Nakamoto and Takayuki Nozawa)

A book-embedding of a graph G is to put the vertices along the spine (a segment) and each edge of G on a single page (a half-plane with the spine as its boundary) so that no two edges intersect transversely in the same page. In this talk, we show that any graph on the projective plane has a book-embedding with seven pages.

Generating even triangulations on surfaces
Naoki Matsumoto (joint work with Atsuhiro Nakamoto and Tsubasa Yamaguchi)

An *even triangulation* on a surface is a map of a simple graph on the surface with each face triangular in which each vertex has even degree. In this paper, we focus on generating theorems for even triangulations on closed surfaces. We first prove that every 4-connected even triangulation on the sphere can be generated from the octahedron by two local transformations, called a 4-splitting and a twin-splitting, preserving the 4-connectivity. Moreover, we also prove that every even triangulation on the torus which might have multiple edges can be generated from one of specified 27 maps or 6-regular toroidal graphs by 4-splittings and twin-splittings.

The PPC is satisfied by 1-deficient oriented graphs with a large girthed strong component
Jean E. Dunbar (joint work with Susan A. van Aardt, Marietjie Frick, and Morten H. Nielsen)

The detour order ($\lambda(D)$ of a digraph D is the order of a longest path in D. A digraph is 1-deficient if its order is exactly one more than its detour order.

The Path Partition Conjecture (PPC) states that if D is any digraph and (a,b) any pair of positive integers such that $a+b = \lambda(D)$, then D has a vertex partition (A,B) such that $\lambda(\langle A\rangle) \leq a$ and $\lambda(\langle B\rangle) \leq b$. We show that the Path Partition Conjecture holds for 1-deficient oriented graphs having a strong component with girth greater than 5.

Perfect digraphs

Stephan Dominique Andres (joint work with Winfried Hochstättler)

We make a first step to generalize the theory of perfect graphs to digraphs. For that purpose we replace the underlying coloring parameter, the chromatic number, by the dichromatic number introduced by Neumann-Lara. As main result we obtain that a digraph is perfect if and only if it does not contain induced directed cycles of length at least 3 and its symmetric part is a perfect graph. Hence, using the Strong Perfect Graph Theorem (SPGT), we derive a characterization of perfect digraphs by means of forbidden induced subdigraphs. We describe some further consequences of the main result in complexity theory and kernel theory.

Proof of a conjecture of Henning and Yeo on vertex disjoint directed cycles

Nicolas Lichiardopol

M.A. Henning and A. Yeo conjectured in 2012 that a digraph of minimum out-degree at least 4, contains 2 vertex disjoint cycles of different lengths. In this paper we prove this conjecture. The main tool, is a new result (to our knowledge) asserting that in a digraph D of minimum out-degree at least 4, there exist two vertex-disjoint cycles C_1 and C_2, a path P_1 from a vertex x of C_1 to a vertex z not in $V(C_1)\cup V(C_2)$, and a path P_2 from a vertex y of C_2 to z, such that $V(P_1)\cap(V(C_1)\cup V(C_2)) = \{x\}$, $V(P_2)\cap(V(C_1)\cup V(C_2)) = \{y\}$, and $V(P_1)\cap V(P_2) = \{z\}$.

Oriented Colourings of Bounded Degree Graphs

Christopher Duffy

By replacing simple graphs with oriented graphs in the homomorphism model of graph colouring we can define an *oriented chromatic number*. Sopena and Vignal conjecture that 7 colours suffice to colour any oriented graph whose underlying simple graph has maximum degree 3. Here we show that such graphs require at most 9 colours, and that oriented graphs whose underlying simple graph have maximum degree 4 require no more than 67 colours in any oriented colouring. Both of these values improve the previous best known upper bound for the oriented chromatic number of these families of graphs.

Mapping planar graphs into Coxeter graph

Qiang Sun (joint work with Ararat Harutyunyan, Reza Naserasr, Mirko Petruševski and Riste Škrekovski)

We conjecture that every planar graph of odd-girth at least 11 admits a homomorphism to the Coxeter graph. Supporting this conjecture, we prove that every planar graph of odd-girth at least 17 admits a homomorphism to the Coxeter graph.

Unique Vector Coloring

Robert Šámal (joint work with David Roberson)

Strict vector coloring is one formulation of an optimization program for Lovász theta function: assign vectors to vertices so that adjacent vertices obtain vectors with large angle in-between. We study when this assignment is unique. We find analog of a classical result on uniqueness of coloring of graph products and generalize it. We use recent result of Laurent and Varvitsiotis on universal rigidity, and ad-hoc exploration of properties of eigenspaces of Kneser graphs.

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