The Structure and Chi-Boundedness of Typical Graphs in a Hereditary Family

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ICGT, Grenoble

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The Structure of My Talk

The Structure of My Talk

- The Beginning

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The Structure of My Talk

- The Beginning
- The Middle
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The Structure of My Talk

- The Beginning
- The Middle
- The End

Part I

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Talking about not talking about the known unknowns.

Bounding The Chromatic Number of Graphs in a Hereditary Family

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χ(G) is the minimum number of colours needed to colour the vertices of G so that no edge is monochromatic.

χ-Boundedness

$\mathcal{F}$ is χ-bounded if there is some function $b_{\mathcal{F}}$ such that for every $G \in \mathcal{F}$ we have:

$$\chi(G) \leq b_{\mathcal{F}}(\omega(G))$$

where $\omega(G)$ is the size of the largest clique in G.

Berge's Perfection

Every graph in $\mathcal{F}$ satisfies χ=ω precisely if $\mathcal{F}$ contains no odd cycle of length at least five and no complement of such a cycle.

Chudnovsky, Robertson, Seymour, Thomas 2006

The χ-Boundness of H-free Graphs

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If H is not a forest then the H-Free graphs are not χ-bounded. *Erdos 1959.*

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Gyarfas' Conjecture: For every forest T, the T-free graphs are χ-bounded. *Gyarfas 1987*

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The Hoang-McDiarmid Conjecture: The vertices of every graph in $\mathcal{F}_2$ can be 2-coloured so that no maximum clique is monochromatic.

Forbidding An Infinite Set of Cycles

Conjecture: For all l, a hereditary family containing no odd cycle of length at least l, is χ-bounded. *Gyarfas 1987*

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Theorem: For all l, a hereditary family containing contains no odd cycle of length at least 5 and no cycle of length at least l is is χ-bounded. *Scott 1999*

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Theorem: If G contains no even induced cycle then $\chi(G) < 2\omega(G)$.
Addarrio-Berry, Chudnovsky,Havet, Reed, Seymour 2008.

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Scott's Example: There are hereditary families which exclude an infinite number of cycles and are not χ-bounded

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For every set of cycles not including C_3 or C_6, almost every graph containing none of these cycles as induced subgraphs satisfies $\chi=\omega$.

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For every H, there is function b_H such that most of the graphs which contain neither H nor its complement as an induced subgraph satisfy $\chi \leq b_H(\omega)$.

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For every H, there is function b_H such that most of the graphs which contain neither H nor its complement as an induced subgraph satisfy $\chi \leq b_H(\omega)$. Furthermore, for almost every H, we can take $b_H(\omega)=o(\omega|V(H)|/\log|V(H)|)$.

Loebl,Reed,Scott,Thomasse,Thomason 2010.

& Kang, Reed, McDiarmid 2014

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For k>5 a.e. C_{2k}-free graph can be partitioned into k-2 cliques and a graph whose complement is the disjoint union of triangles and stars.
Reed and Scott 2014

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A *witnessing partition of H-freeness* is a partition of the vertex set of G into $S_1,\ldots,S_{wpn(H)}$ such that for any partition of $V(H)$ into $X_1,\ldots,X_{wpn(H)}$ there is an i for which $H[X_i]$ is not an induced subgraph of $G[S_i]$.

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Conjecture: For all *H*, almost every *H*-free graph has a witnessing partition of *H*-freeness.

Triangle Free Graphs

There are least $2^{n^2/4}$ bipartite graphs on {1,..,n}.

Coloured Regularity Partition Cliques

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For an equi-partition X_1,…,X_p of V(G)= {1,…,n}, an edge ij of the clique on {1,…,p} is:

Grey if (X_i,X_j) is irregular,
Blue if (X_i,X_j) is regular with density near 0,
Green Otherwise

Every large enough G has a partition with $o(p^2)$ grey edges for p as large as we want but fixed. *Szemeredi*

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If G is Δ-free there is no triangle whose edges are green.

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If G is Δ-free there is no triangle whose edges are green.
Hence at most half the edges of the edges are green.

Triangle-Free Graphs: A Rough Partition

The number of graphs permitting a partition with $p_g\binom{p}{2}$ green edges is $2^{(1+o(1))p_g\binom{n}{2}}$

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The associated green graph is Δ-free, so contains a bipartite graph with $\frac{(1+o(1))}{2}\binom{p}{2}$ edges.

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This provides a bipartition of {1,..,n} with $o(n^2)$ edges of G within each side.

Triangle-Free Graphs: Refining The Partition

For each partition of {1,...,n} into A and B there are $2^{o(n^2)}$ choices for subgraphs G[A] and G[B] with $o(n^2)$ edges. For each such choice there are at most $2^{|A||B|}$ choices for E(A,B).

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(ii) A contains a stable set of size |A|(1-o(1)),

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Coloured Regularity Partition Cliques Revisited

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Green Otherwise

If G is H-free there is no injection f:V(H)->{1,…,p} such that for each edge uv of H, f(u)f(v) is green or red, and for each non-edge uv f(u)f(v) is green or blue.

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This implies that there is no green wpn(H)+1 clique in a coloured partition clique corresponding to G.

H-free Graphs: A Rough Partition

For some c+s=wpn(H), there are $2^{(1-\frac{1}{wpn(H)}+o(1))\binom{n}{2}}$ H-free graphs on {1,…,n} whose vertices can be partitioned into c cliques and s stable sets.

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The vertices of a.e. H-free graph on {1,…,n} can be equipartitioned into wpn(H) sets such that each partition class permits a Szemeredi partition permitted by $2^{o(\binom{n}{2})}$ graphs.

A Better Rough Partition

(*Reed,Scott* cf. *Alon,Balogh,Bollobas,Morris)*

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For every H,δ there are ε,b>0 s.t. for a.e. H-free graph, V(G) has a partition into $S_1,\ldots,S_{wpn(H)}$ for which there are two exceptional sets Z and B with $|Z|\leq n^{2-\varepsilon}$ and $|B|\leq b$ such that:

(i) $(S_1 - Z,\ldots,S_{wpn(H)} - Z)$ is an H-witnessing partition of G-Z

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(ii) for every v in S_i there is a w in B such that:

$$|S_i \cap ((N(v) - N(w)) \cup (N(w) - N(v))| \leq \delta n$$

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This implies that for every S_i, there are bipartitite, split, and cobipartite subgraphs of H which are not induced subgraphs of $G[S_i - Z]$. Hence the number of choices for $G[S_i]$ is at most 2 to the $n^{2-\varepsilon.}$

More on This Partition

(*Kang,McDiarmid,Reed)*

For every δ, for almost every H for all but (i) (S_1 - Z,...,$S_{wpn(H)}$-Z) is an H-witnessing partition of G-Z in which o(wpn(H)) of the S_i-Z are obtained from a split graph by substituting stable sets for the vertices of the clique and cliques for the vertices of the stable set,

Possible Partitions for C_6-Free Graphs

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Otherwise, since not both S_1-Z and S_2-Z contain a copy of F if F is P3, its complement or a stable set of size 3, one is the disjoint union of cliques the other is complete multipartite and either the first has only two cliques or the second is the complement of a matching.

Typical C_6-Free Graphs

Almost every C_6-free graph can be partitioned into a stable set and the complement of a graph of girth 5.

Almost every C_6-free graph satisfies:

$$\chi(G)=O(\omega(G)^2).$$

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(c) If l is 3 and we forbid no even cycle then almost every graph in $\mathcal{F}$ satisfies $\chi=\omega=2$.
(d) If we forbid cycles of length 3 and 4 and allow only a finite number of cycle lengths then every graph in $\mathcal{F}$ satisfies $\chi=O(\omega)$ *Randerath and Schiermeyer 2001.*

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(d) If we forbid cycles of length 3 and 4 and allow only a finite number of cycle lengths then every graph in $\mathcal{F}$ satisfies $\chi=O(\omega)$ *Randerath and Schiermeyer 2001.*
(e) for other sets where l is 3, the situation is less clear.

Further Work

Disproving The Conjecture

Determining the structure of the graphs induced by the partition classes in typical H-free graphs for specific H and typical H.

Characterize completely the structure of typical graphs when we forbid a triangle and some other set of cycles, including at least one even one.

Thank you!