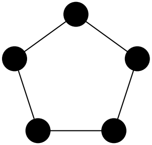
Combinatorial Reconfiguration

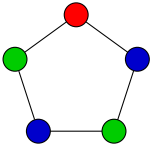
Marthe Bonamy

November 15, 2018

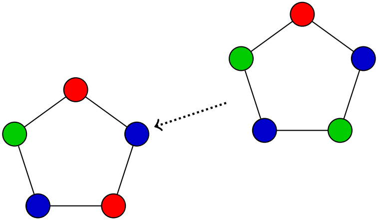
Graph



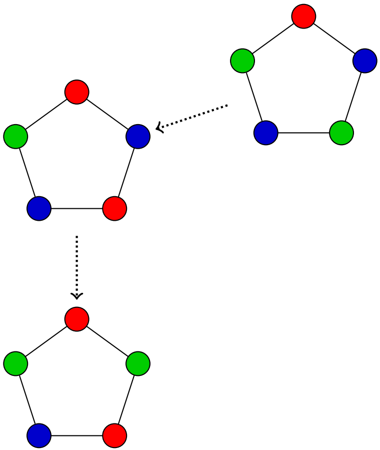
Graph coloring



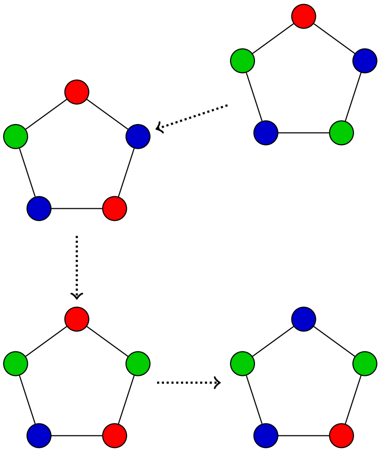
Graph recoloring



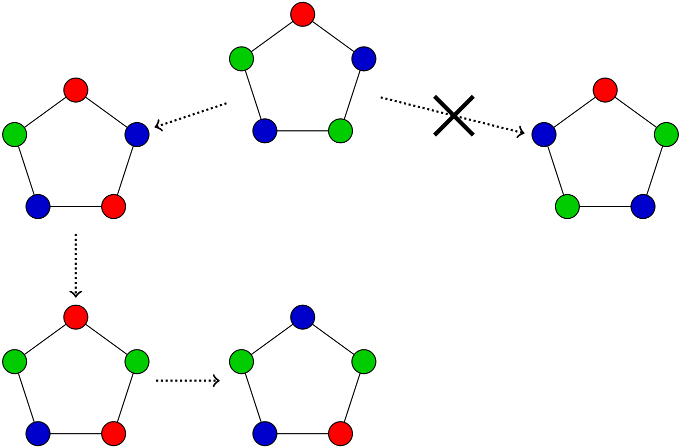
Graph recoloring



Graph recoloring

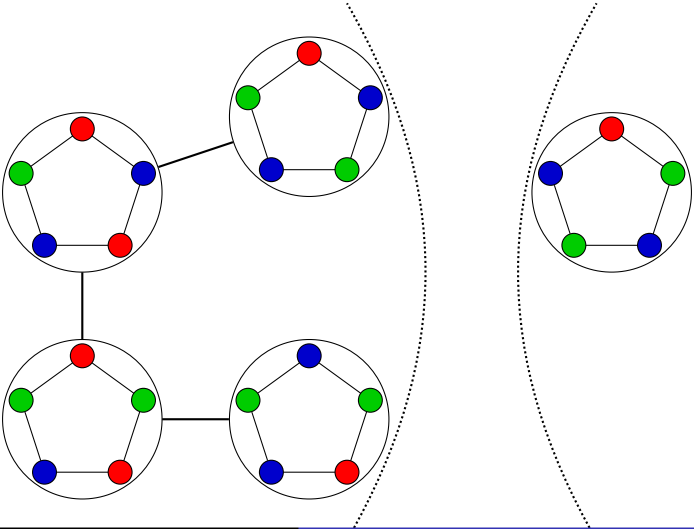


Graph recoloring

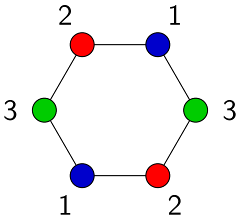


Graph recoloring ⇒ Reconfiguration graph

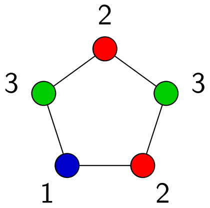
Solutions // Nodes. Most similar solutions // Neighbors.



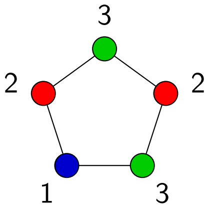
Graph 3-recoloring: bad cases (1)



Graph 3-recoloring: bad cases (2)

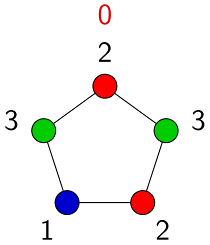


α

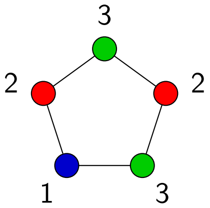


β

Graph 3-recoloring: bad cases (2)

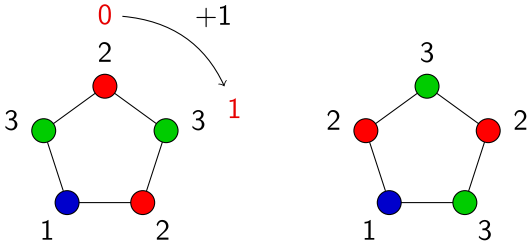


α



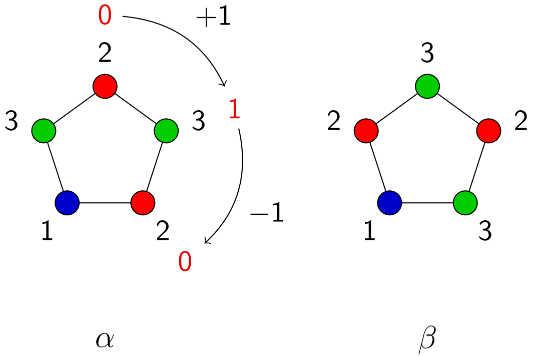
β

Graph 3-recoloring: bad cases (2)

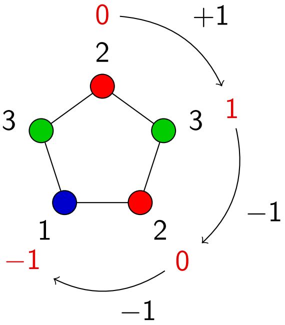


α β

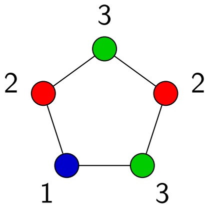
Graph 3-recoloring: bad cases (2)



Graph 3-recoloring: bad cases (2)

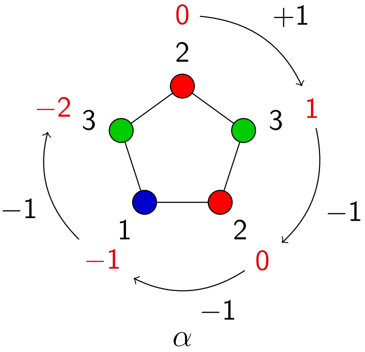


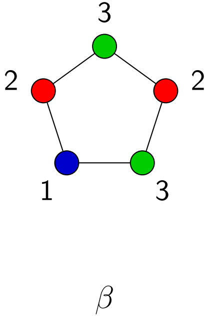
α



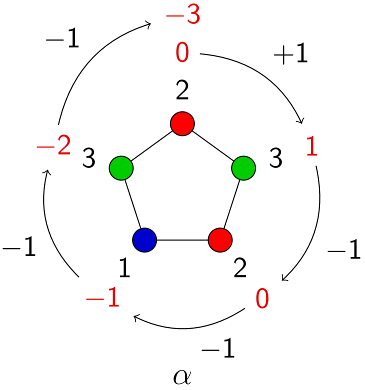
β

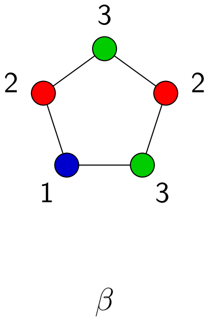
Graph 3-recoloring: bad cases (2)



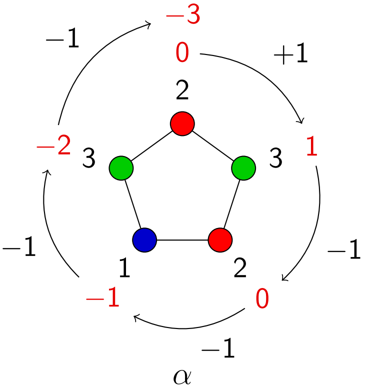


Graph 3-recoloring: bad cases (2)

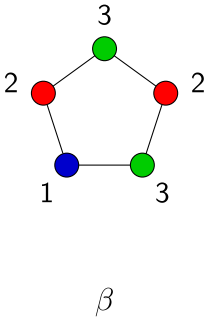




Graph 3-recoloring: bad cases (2)



$w(\alpha, C_5) = -3$



$w(\beta, C_5) = 3$

Graph recoloring: cycles of length multiple of 3

Theorem (Cereceda, Johnson, van den Heuvel '11)

For any graph G, any two 3-colourings α, β, if

- *Neither α nor β contain a frozen cycle, and*
- *α and β have the same wrapping number on every cycle,*

then G can be recoloured from α to β.

Graph recoloring: cycles of length multiple of 3

Theorem (Cereceda, Johnson, van den Heuvel '11)

For any graph G, any two 3-colourings α, β, if

- *Neither α nor β contain a frozen cycle, and*
- *α and β have the same wrapping number on every cycle,*

then G can be recoloured from α to β.

Corollary (Wrochna '15)

All graphs with no cycle of length multiple of 3 are 3-colourable.

Graph recoloring: induced cycles of length multiple of 3

Conjecture (Folklore '15)

Every graph with no induced cycle of length multiple of 3 contains an edge whose removal does not create an induced cycle of length multiple of 3.

Graph recoloring: induced cycles of length multiple of 3

Conjecture (Folklore '15)

Every graph with no induced cycle of length multiple of 3 contains an edge whose removal does not create an induced cycle of length multiple of 3.

Hypothetical Corollary (Wrochna '15)

All graphs with no induced cycle of length multiple of 3 are 3-colourable.

Theorem (Bonamy, Charbit, Thomassé '15)

All graphs with no induced cycle of length multiple of 3 are $O(1)$-colourable.

Graph recoloring: induced cycles of length multiple of 3

Conjecture (Folklore '15)

Every graph with no induced cycle of length multiple of 3 contains an edge whose removal does not create an induced cycle of length multiple of 3.

Nope! (Wrochna '18)

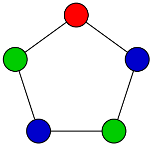
Hypothetical Corollary (Wrochna '15)

All graphs with no induced cycle of length multiple of 3 are 3-colourable.

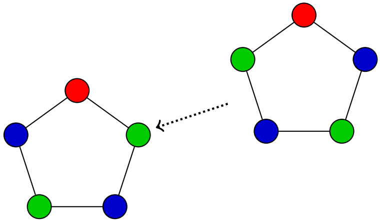
Theorem (Bonamy, Charbit, Thomassé '15)

All graphs with no induced cycle of length multiple of 3 are $O(1)$-colourable.

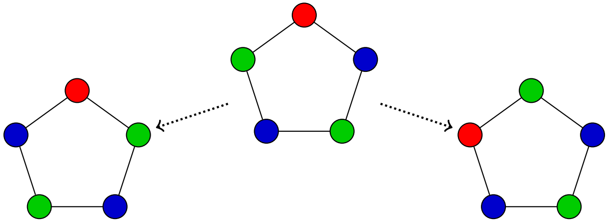
Kempe equivalence



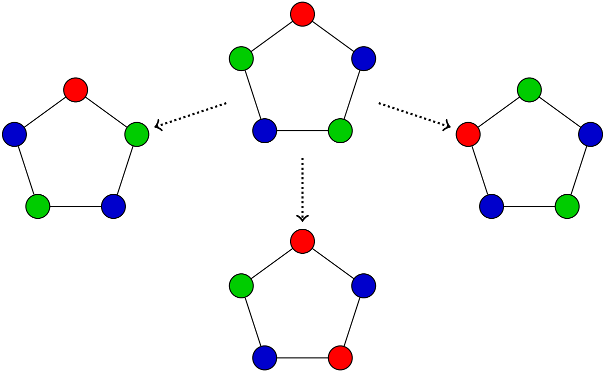
Kempe equivalence



Kempe equivalence



Kempe equivalence



Kempe equivalence: Goal

Δ: Maximum degree of the graph

Kempe equivalence: Goal

Δ: Maximum degree of the graph

Theorem (Brooks '41)

Every graph is Δ-colourable, except for cliques and odd cycles.

Kempe equivalence: Goal

Δ: Maximum degree of the graph

Theorem (Brooks '41)

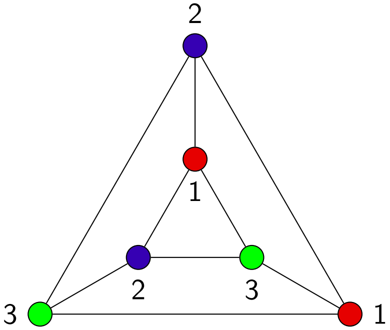
Every graph is Δ-colourable, except for cliques and odd cycles.

Conjecture (Mohar '05)

All the Δ-colourings of a graph are Kempe equivalent.

Kempe equivalence: Results

The conjecture is false! (van den Heuvel '13)



(3-prism)

Kempe equivalence: Results (2)

Theorem (Feghali, Johnson, Paulusma '15)

True for all graphs with $\Delta \leq 3$ (other than the 3-prism).

Kempe equivalence: Results (2)

Theorem (Feghali, Johnson, Paulusma '15)

True for all graphs with $\Delta \leq 3$ (other than the 3-prism).

Theorem (B., Bousquet, Feghali, Johnson '15)

True for all graphs (other than the 3-prism).

Kempe equivalence: Results (2)

Theorem (Feghali, Johnson, Paulusma '15)

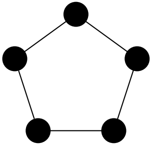
True for all graphs with $\Delta \leq 3$ (other than the 3-prism).

Theorem (B., Bousquet, Feghali, Johnson '15)

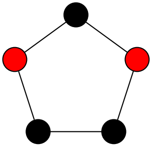
True for all graphs (other than the 3-prism).

Understand Glauber Dynamics (analyse Antiferromagnetic Potts Model when the temperature tends to 0)

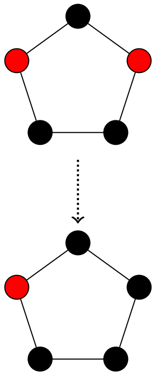
Independent Set Reconfiguration



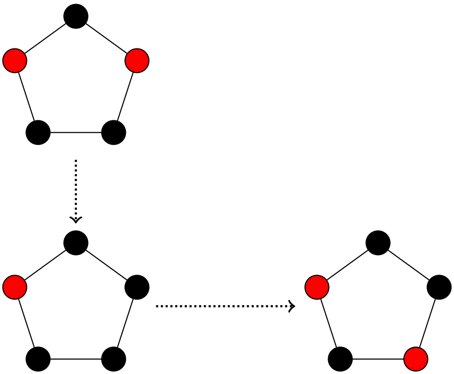
Independent Set Reconfiguration



Independent Set Reconfiguration

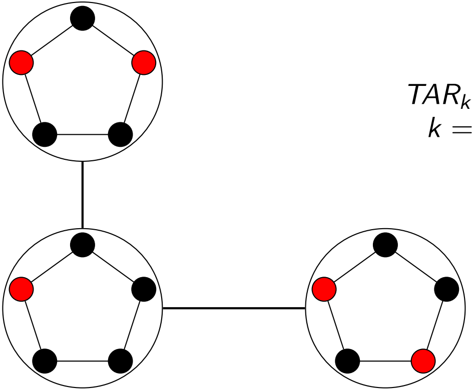


Independent Set Reconfiguration



Independent Set Reconfiguration ⇒ Reconfiguration Graph

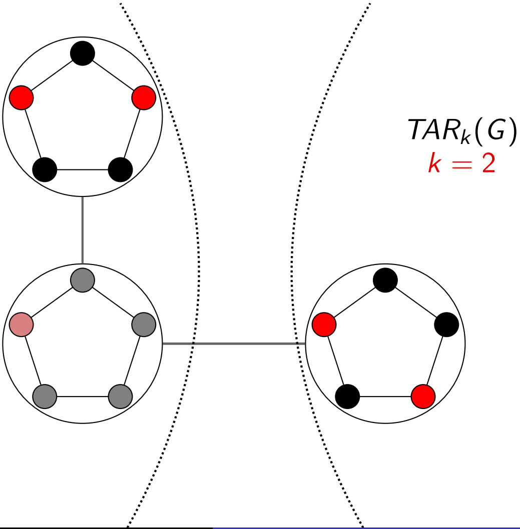
Solutions // Vertices. Closest solutions // Neighbors.



$TAR_k(G)$
$k = 1$

Independent Set Reconfiguration ⇒ Reconfiguration Graph

Solutions // Vertices. Closest solutions // Neighbors.



- Two solutions:
  - In the same connected component?
  - What distance between them?

Reconfiguration Graphs

- Two solutions:
  - In the same connected component?
  - What distance between them?
- Reconfiguration graph:
  - Connected?
  - Maximal diameter of a connected component?

Reconfiguration Graphs

- Two solutions:
  - In the same connected component?
  - What distance between them?
- Reconfiguration graph:
  - Connected?
  - Maximal diameter of a connected component?

Various problems, various elementary steps...

Elementary steps

1. Token Addition and Removal (TAR): We can *add* or *remove* tokens (up to some cardinality constraints)

Elementary steps

1. Token Addition and Removal (TAR): We can *add* or *remove* tokens (up to some cardinality constraints)
2. Token Jumping (TJ): A token *jumps* to anywhere else

Elementary steps

1. Token Addition and Removal (TAR): We can *add* or *remove* tokens (up to some cardinality constraints)
2. Token Jumping (TJ): A token *jumps* to anywhere else
3. Token Sliding (TS): A token *slides* along an edge

Elementary steps

1. Token Addition and Removal (TAR): We can *add* or *remove* tokens (up to some cardinality constraints)
2. Token Jumping (TJ): A token *jumps* to anywhere else
3. Token Sliding (TS): A token *slides* along an edge

Reconfiguration graph connected $\Rightarrow$ Efficient enumeration?

Elementary steps

1. Token Addition and Removal (TAR): We can *add* or *remove* tokens (up to some cardinality constraints)
2. Token Jumping (TJ): A token *jumps* to anywhere else
3. Token Sliding (TS): A token *slides* along an edge

Reconfiguration graph connected $\Rightarrow$ Efficient enumeration? Sampling?

Elementary steps

1. Token Addition and Removal (TAR): We can *add* or *remove* tokens (up to some cardinality constraints)
2. Token Jumping (TJ): A token *jumps* to anywhere else
3. Token Sliding (TS): A token *slides* along an edge

Reconfiguration graph connected $\Rightarrow$ Efficient enumeration? Sampling?

Almost every thing is PSPACE-hard.
$\Rightarrow$ Restricted graph classes, Fixed Parameter Tractability

Conclusion

- Distributed recolouring (with Paul Ouvrard, Mikaël Rabie, Jukka Suomela, Jara Uitto, DISC'2018)
- Distributed reconfiguration of independent sets (Keren Censor-Hillel and Mikaël Rabie, 2018)

Conclusion

- Distributed recolouring (with Paul Ouvrard, Mikaël Rabie, Jukka Suomela, Jara Uitto, DISC'2018)
- Distributed reconfiguration of independent sets (Keren Censor-Hillel and Mikaël Rabie, 2018)
- Stronger links with applications
- Meta-theorems

Conclusion

- Distributed recolouring (with Paul Ouvrard, Mikaël Rabie, Jukka Suomela, Jara Uitto, DISC'2018)
- Distributed reconfiguration of independent sets (Keren Censor-Hillel and Mikaël Rabie, 2018)
- Stronger links with applications
- Meta-theorems

Merci !